GPIF working paper

Bond investment strategy based on factor analysis: Revisited
Application to carry and roll-down strategy in the U.S. Treasury market

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EXECUTIVE SUMMARY

- As one of the traditional bond investment strategy, there is "carry and roll-down strategy", which assumes that the future yield curve remains unchanged.
- In the U.S. Treasury market, where the volatility of yields is reasonably high, it is mentioned that "the assumption that future yield curves will remain unchanged is hard to be satisfied".
- This study revisited Litterman and Scheinkman [1991] for the first time in 30 years to explore the potential of an improved carry and roll-down strategy in the U.S. Treasury market, adding scenario returns to carry and roll-down strategy based on the factor analysis.

1. Introduction

1.1 Literature Review

Factor analysis is one method of multivariate analysis, which is a statistical analysis method. According to Noguchi (2018), "a method of explaining information with many variables by a small number of implicit common factors." Factor analysis is often used in the field of social sciences, for example in the field of education, to express the performance of various courses in schools by weighting implicit common factors such as logical thinking, memory, and computational capability, and in the field of marketing, to extract factors concerning consumer preferences from each indicator of purchasing behavior. In addition to factor analysis, principal component analysis is also a statistical analysis technique similar to factor analysis among multivariate analysis. Principal components analysis differs from factor analysis in that principal components are determined in descending order of their contribution to the variance of observed data without placing common factors. Putting strict definition aside, we regard factor analysis as principal component analysis in the sense of the statistical method and review preceding researches applied to bond market analysis in this chapter.

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To best of our knowledge, Litterman and Scheinkman [1991] is the first paper utilizing factor analysis to shed some light on bond market. They applied factor analysis to changes in U.S. zero-coupon government bond yields, decomposed the changes in yields into three factors, and showed the impact (derived from factor loading of each factor) of each factor on the yield curve with the remaining maturity as the horizontal axis and the change in yields as the vertical axis. From the shape of the impact of each factor on the maturity of government bonds, the first factor is called the "level" factor, the second factor is called the "slope" factor, and the third factor is called the "curvature" factor. They showed that approximately 98% of the yield movement could be explained by these three factors. In addition, they proposed a method for hedging the risk of the yield curve using the sensitivity of each factor to various government bonds.

Since Litterman and Scheinkman [1991] was published, the application of factor analysis to bond market analysis have been extensively studied by financial institutions. However, such kinds of studies applied to Japanese bond market analysis is quite limited, Hayashida (2005), Hoshika and Miyazaki (2007), and Sakudo (2010) are scarce examples.

Hayashida (2005) generated the future return paths of fixed-rate government bonds, floating-rate government bonds, and short-term assets, and determined expected returns and risks of those bonds, by way of three-factor model that used principal component analysis for the changes in spot rates in constructing models of yield curve movements. Hoshika and Miyazaki (2007) discussed an effective bond investment strategy based on the forecast of changes in bond yields using a two-factor model, considering the speed of yield changes and the trade-off between trading opportunities and transaction costs, which were overlooked in Reisman and Zohar (2004 a, 2004 b, 2004 c). Sakudo (2010) confirmed a linear relationship between forward rates and excess returns in the Japanese government bond market via tent-type regression coefficients similar to those of Cochrane and Piazzesi (2005). Moreover, he applied factor analysis to the forward rates in the Japanese government bond market to evaluate the three factors: the level, slope, and curvature, and found that the curvature factor had the greatest impact on the risk premium of bond investment.

Although “carry and roll-down strategy” is a bit old-fashioned bond investment strategy, it has regained popularity since the early stage of the quantitative easing by the Bank of Japan. Yamada (2000) is the pioneering literature to provide an essential discussion of carry and roll-down strategy for the Japanese government bond market. Yamada (2000) calculated the excess return in carry and roll-down strategy from the government bond index for the period from February 1988 to September 2000 and concluded that “the average excess return is high at 0.88% and the sharp ratio adjusted for tracking error (=0.72%) is also quite high at 1.2.” Yamada (2000) also found that the optimal portfolio of carry and roll-down strategy was bullet-type except for just a brief period of time, and gave up convexity relative to the government bond index, but he analyzed that the excess return was remained due to the small negative convexity to the index coming from changes in yields over the same period.

In recent years, Kikugawa et al. (2017) identified carry and roll-down strategy as one factor and mentioned that its premium is large and its correlation with credit and stocks is low. They noted consequently the investment in this factor improves the performance of bond investments. In addition, they presented "The central bank has announced a policy of intervening in the shape of the yield curve. If it makes the curve more skewed, the effect of the carry and roll-down factor will be enhanced."

In contrast, in the U.S. Treasury market where the yield level is relatively high even in the global low-yield environment (please refer to Kimura (2021) for more detailed analysis), volatility fluctuates considerably by reflecting the monetary policy of the U.S. Federal Reserve System (Fed), and it is difficult to satisfy the condition that the future yield curve is unchanged, which carry and roll-down strategy relies on.

1.2 The purpose of this study and the organization of this paper

In this study, we first conduct an empirical analysis to what extent excess returns, tracking errors, and information ratios could be obtained when using the U.S. Treasury index as a benchmark for carry and roll-down strategy in the U.S. Treasury market. The analysis here is long-only, therefore it differs from the analysis by Koijen et al. (2015) that compared carry trades which included shorting U.S. Treasuries, to an equally weighted benchmark. To the best of our knowledge, there is no studies that attempt to apply factor analysis to carry and roll-down strategy. Therefore, the purpose of this study is to revisit Litterman and Scheinkman [1991] for the first time in 30 years to explore effective applications of factor analysis to carry and roll-down strategy in order to improve the performance of carry and roll-down strategy in the U.S. Treasury market.

The organization of this study is as follows. In Chapter 2, we review returns, expected returns, and returns under the yield curve unchanged scenario on bond investments, and analyze the performance of carry and roll-down strategy in the U.S. Treasury market. In Chapter 3 we report the results of applying factor analysis to U.S. Treasury yields. In Chapter 4 we report on the effective application of factor analysis to U.S. Treasury yields to carry and roll-down strategy considering the results of Chapter 3 and examples of the application of this approach. In the final chapter we summarize the research and clarify future challenges.

2. Performance Analysis of carry and roll-down strategy in the U.S. Treasury market

2.1 Returns, expected returns, and returns under the yield curve unchanged scenario on government bonds

We illustrate the return, expected return, and return under the yield curve unchanged scenario of holding a 5-year bond for 0.5 years and selling it with a coupon, as an example here. $Y_{0,5}, P_5(Y_{0,5})$ express the yield and price of the 5-year bond at time zero (assuming that the start of the investment is time zero), respectively. These are the values that are fixed at time zero. At the end of 0.5 years, the 5-year bond becomes a 4.5-year bond, at which time half of the coupon (C_5) on the 5-year bond is paid. $\tilde{Y}_{0.5,4.5} P_{4.5}(\tilde{Y}_{0.5,4.5})$ express the yield and price of a 4.5-year bond in 0.5 years past, respectively. We place tiled (~) above to show that these are random variables that are not fixed at time zero. The expected value at time zero for the yield on a 4.5-year bond in 0.5 years past is equal to the forward yield $F_{0,0.5,4.5}$ on a 4.5-year bond starting in 0.5 years from time zero. Here, $P_{0,5}(x), P_{0.5,4.5}(x)$ represents the following functions.

$$P_{0,5}(x) = \frac{C_5/2}{\left(1+\frac{x}{2}\right)^1} + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^2} + \cdots + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^9} + \frac{C_5/2+100}{\left(1+\frac{x}{2}\right)^{10}}$$

$$P_{0.5,4.5}(x) = C_5/2 + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^1} + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^2} + \cdots + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^8} + \frac{C_5/2+100}{\left(1+\frac{x}{2}\right)^9}$$

The return, expected return, and carry-roll-down over the next six months from time zero could be derived by Taylor expansion of the above functions to a second-order term in the vicinity of the current 5-year bond yield $Y_{0,5}$, then reformed into a simple equation. (see Appendix for return)

Return:

$$\tilde{R}_{5\to4.5} = \frac{P_{0.5,4.5}(\tilde{Y}_{0.5,4.5}) - P_{0,5}(Y_{0,5})}{P_{0,5}(Y_{0,5})} \approx \frac{1}{P_{0,5}(Y_{0,5})}\Big[\frac{C_5}{2} + P'_{0.5,4.5}(Y_{0,5})(Y_{0,4.5}-Y_{0,5}) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(Y_{0,4.5}-Y_{0,5})^2 + P'_{0.5,4.5}(Y_{0,5})(\tilde{Y}_{0.5,4.5}-Y_{0,4.5}) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(\tilde{Y}_{0.5,4.5}-Y_{0,4.5})^2\Big] \quad (1)$$

In square brackets, the first term represents carry profit, the second term (static duration) and the third term (static convexity) together represent roll-down profit, and the fourth term (dynamic duration) and the fifth term (dynamic convexity) together represent capital gains and losses other than roll-down, respectively.

Expected returns:

$$E_0(\tilde{R}_{5\to4.5}) = E_0\left(\frac{P_{0.5,4.5}(\tilde{Y}_{0.5,4.5}) - P_{0,5}(Y_{0,5})}{P_{0,5}(Y_{0,5})}\right) \approx \frac{1}{P_{0,5}(Y_{0,5})}\Big[\frac{C_5}{2} + P'_{0.5,4.5}(Y_{0,5})(Y_{0,4.5}-Y_{0,5}) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(Y_{0,4.5}-Y_{0,5})^2 + P'_{0.5,4.5}(Y_{0,5})(F_{0,0.5,4.5}-Y_{0,4.5}) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(\sigma_{4.5})^2\Big] \quad (2)$$

Here $\sigma_{4.5}$ expressed the volatility of the semiannual yield change on a 4.5-year bond.

Carry and roll-down (defined as the return under the yield curve unchanged scenario, hereafter CA&RD):

$$CA\&RD_{5\to4.5} \approx \frac{1}{P_{0,5}(Y_{0,5})}\Big[\frac{C_5}{2} + P'_{0.5,4.5}(Y_{0,5})(Y_{0,4.5}-Y_{0,5}) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(Y_{0,4.5}-Y_{0,5})^2\Big] \quad (3)$$

Under the yield curve unchanged scenario, $\tilde{Y}_{0.5,4.5} = Y_{0,4.5}$, and the capital gain and loss other than the roll-down in parentheses in the return equation (1) are set at 0.

2.2 Carry and roll-down strategy

We divided the U.S. Treasury market to which the investment is made into 13 buckets according to the remaining period as follows, and named bucket 1 to bucket 13 in order; 1 year (remaining maturity 1 year or less), 2 years (remaining maturity more than 1 year and 2 years or less), 3 years (remaining maturity more than 2 years and 3 years or less), 4 years (remaining maturity more than 3 years and 4 years or less), 5 years (remaining maturity more than 4 years and 5 years or less), 6 years (remaining maturity more than 5 years and 6 years or less), 7 years (remaining maturity more than 6 years and 7 years or less), 8 years (remaining maturity more than 7 years and 8 years or less), 9 years (remaining maturity more than 8 years and 9 years or less), 10 years (remaining maturity more than 9 years and 10 years or less), 15 years (remaining maturity more than 10 years and 15 years or less), 20 years (remaining maturity more than 15 years and 25 years or less), 30 years (remaining maturity more than 25 years and 30 years or less)

We denoted respectively the market weights of each bucket by $w_1^M \cdots w_{13}^M$, the durations of each bucket by $D_1^M \cdots D_{13}^M$, the total market weight by 1 and the duration of the market as a whole by D (in this study, we used FTSE U.S. Treasury Bond Index as a benchmark representing the market as a whole). Also, we denoted carry and roll-down for each bucket by $CA\&RD_{1\to 0.5}, \cdots, CA\&RD_{13\to(13-0.5)}$. The subscript "1-10" indicates the period, 11 represents 15 years, 12 represents 20 years and 13 represents 30 years, and -0.5 in parentheses represents that a half year has passed in the bucket). Here, in order to ensure that the excess returns from carry and roll-down strategy do not depend on the direction of the market, we made constraints that the duration of the portfolio and the duration of the benchmark are matched when attempting to maximize carry and roll-down (the weights of each bucket $w_1, \cdots, w_{13}$ are determined so that carry and roll-down of the portfolio becomes maximum). The optimization model and excess return (α), which express carry and roll-down strategy, are as follows:

[Optimization model]

Maximize the objective function: $\underset{w_1, \cdots w_{13}}{Max} CA\&RD_{1\to 0.5} \cdot w_1 + \cdots CA\&RD_{13\to(13-0.5)} \cdot w_{13}$

Constraints: $D_1^M \cdot w_1 + \cdots + D_{13}^M \cdot w_{13} = D$

: $w_1 + \cdots + w_{13} = 1$

: $w_1 \geq 0, \cdots, w_{13} \geq 0$

$w_1^*, \cdots, w_{13}^*$ denotes optimal weights obtained by implementing the optimization model.

[Excess gain and loss on carry and roll-down strategy (α)]

$$\alpha = \tilde{R}_{1\to 0.5}(w_1^* - w_1^M) + \cdots + \tilde{R}_{13\to(13-0.5)}(w_{13}^* - w_{13}^M) \tag{4}$$

Here we confirm the excess gain and loss (α) of carry and roll-down strategy. α is a random variable and takes both positive and negative values. When it is positive it becomes excess gain, and when it is negative it becomes excess loss. If the yield curve unchanged scenario is realized six months later, the return on each government bond $\tilde{R}_{i\to(i-0.5)}$ becomes $CA\&RD_{i\to(i-0.5)}$ whose maturity is changed from each maturity of $CA\&RD_{5\to 4.5}$ given in equation (3) in the case of the 5-year bond, therefore α is to be $\alpha_{CA\&RD}$ as follows.

$$\alpha_{CA\&RD} = CA\&RD_{1\to 0.5}(w_1^* - w_1^M) + \cdots + CA\&RD_{13\to(13-0.5)}(w_{13}^* - w_{13}^M) \tag{5}$$

Since $w_1^*, \cdots, w_{13}^*$ in equation (5) is the optimal weight that maximizes the objective function under the yield curve unchanged scenario, it always exceeds the value of the objective function using the market weight $w_1^M \cdots w_{13}^M$. Consequently, $\alpha_{CA\&RD}$, is always the positive value and becomes the excess return. Thus, whether carry and roll-down strategy produces excess gain and loss (α) depends on the feasibility of the yield curve unchanged scenario.

In terms of the Japanese government bond market, as the Bank of Japan's quantitative and qualitative monetary easing with yield curve control has been pervasive, the volatility of the yield curve is small, and the realization of the yield curve unchanged scenario is highly likely. Therefore, as Kikugawa et al. (2017) point out, carry and roll-down strategy could improve the performance of bond investments. On the other hand, the U.S. Treasury market is in an environment in which the Fed is expected to raise interest rates in the future at the time of writing. In the U.S. Treasury market as well, as pointed out by Yamada (2000), which analyzed the Japanese government bond market, it is assumed that the optimal portfolio of carry and roll-down strategy will often become bullet-type. However, in an environment where the Fed is expected to raise interest rates going forward, it is expected that an upward curvature will appear in the medium-term maturity buckets, and bullet-type performance will be relatively weakened. In addition, the U.S. Treasury market is much efficient than the Japanese government bond market, and it is possible that the expected profit from carry and roll-down could be easily eliminated due to the dynamics of the yield curve. We therefore examine the performance of carry and roll-down strategy in the U.S. Treasury market.

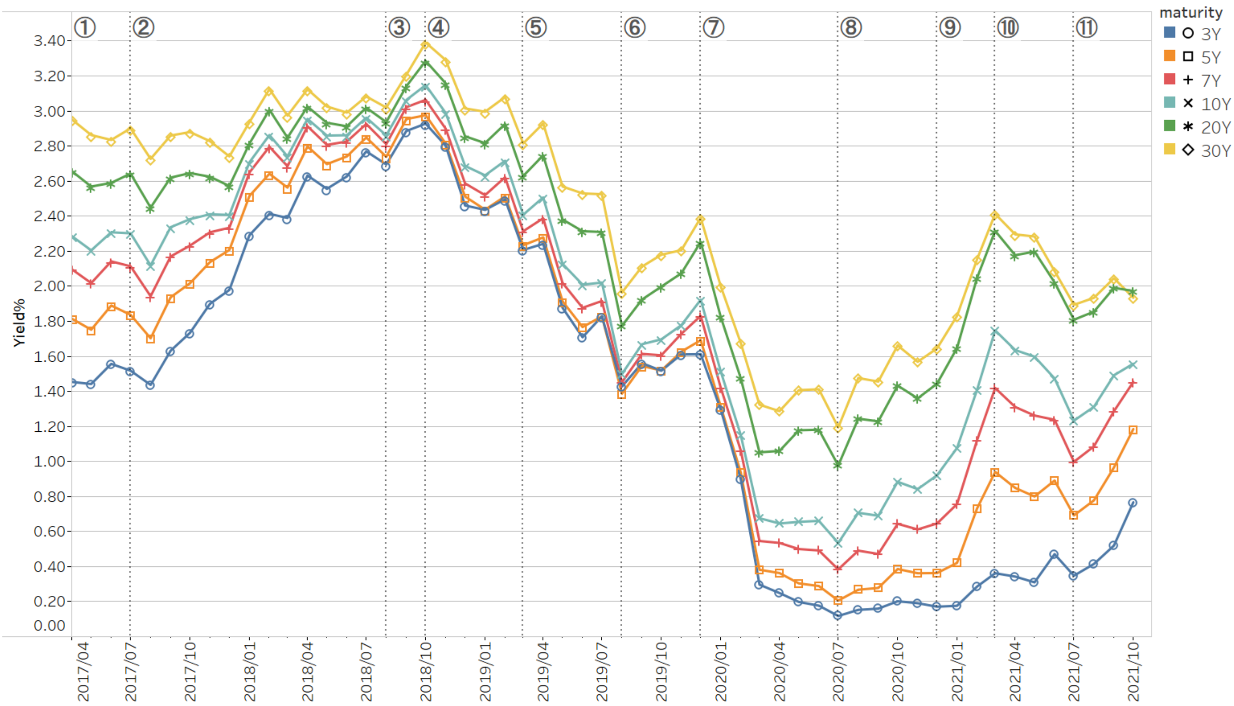
2.3 Performance Analysis of carry and roll-down strategy in the U.S. Treasury market

This Chapter attempts the performance analysis of carry and roll-down strategy using U.S. Treasury yields (constant maturity par yields) for approximately four and a half years from the end of April 2017 to the end of October 2021. The estimation period is set from the end of April 2017 to the end of March 2019 since the estimation period for the model (factors and factor loading) is required in Chapter 3 and 4. The performance analysis is conducted covering over the period of the out-of-sample from the end of March 2019 to the end of October 2021. Firstly, at the end of March 2019, the U.S. Treasury market is divided into 13 buckets, as described in Section 2.2 and we calculate the optimal weights that maximize carry and roll-down. CA&RD return in April 2019 is given by the return on investment with these weights for a month. Then we compare the benchmark returns with CA&RD returns calculated in the same way for October 2021.

To start with the analysis, we review the dynamics of U.S. Treasury yields over the period covered by the analysis. Figure 1 shows the yield movements for 3-year bond, 5-year bond, 7-year bond, 10-year bond, 20-year bond, and 30-year bond. Because carry and roll-down strategy generally adopts bullet or barbell-type portfolios, performance could be inferred by the 5-year and 7-year yield relative to the combination of 3-year and 30-year yield. Firstly, we review the dynamics of the yield curve from the end of April 2017 to the end of March 2019, which is the estimation period for the model in Chapter 3 and 4. This period, even though it is only two years, consists of variety of yield regimes such as ① the flattening rally from the end of April 2017 to the end of July 2017, ② the flattening sell-off from the end of July 2017 to the end of August 2018, ③ the steepening sell-off from the end of August 2018 to the end of October 2018, and ④ the steepening rally from the end of October 2018 to the end of March 2019.

Next, we review the period from the end of March 2019 to the end of October 2021 covered by this performance analysis in this Chapter. This period consists of the following periods; ⑤ parallel rally from the end of March 2019 to the end of August 2019, ⑥ modest steepening sell-off from the end of August 2019 to the end of December 2019, ⑦ steepening rally from the end of December 2019 to the end of July 2020, ⑧ modest steepening sell-off from the end of July 2020 to the end of December 2020, ⑨ intense steepening sell- off from the end of December 2020 to the end of March 2021, ⑩ flattening rally from the end of March 2021 to the end of July 2021, ⑪ flattening sell-off from the end of July 2021 to the end of October 2021.

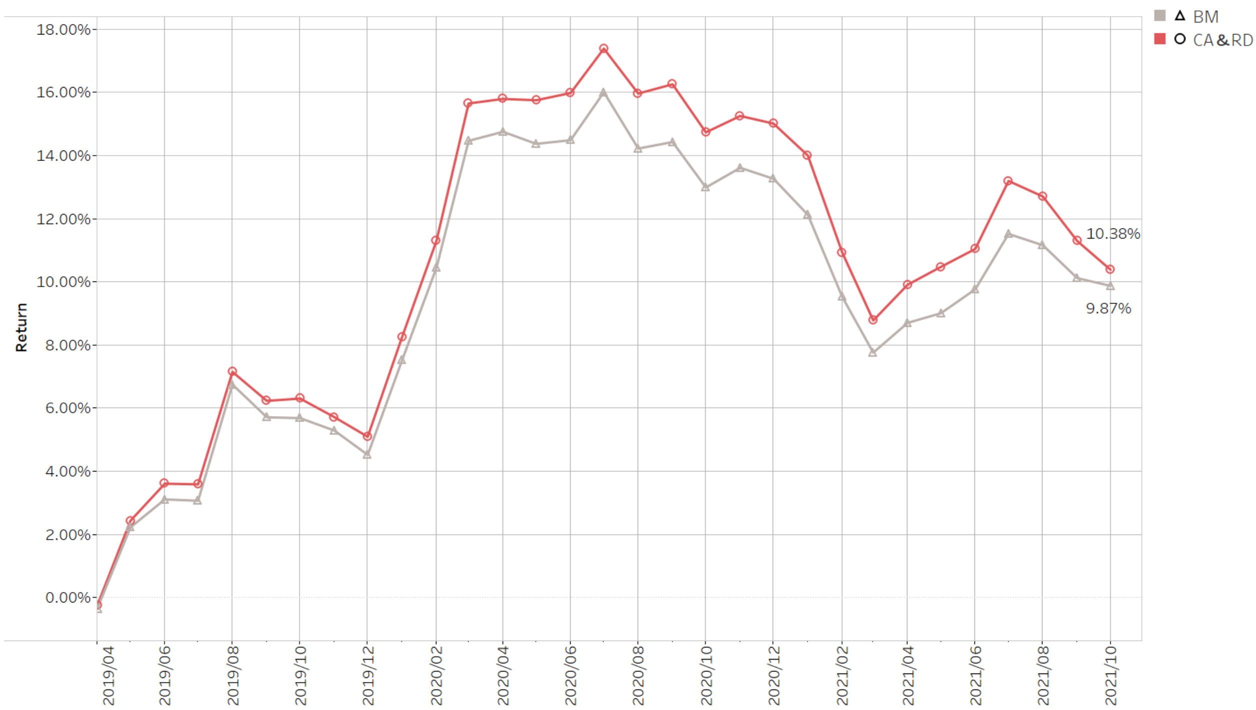
Figure 1: Trends of the U.S. Treasury yields (constant maturity per yield, in percentage)



(Source: Prepared by the authors from FactSet data)

Figure 2 shows the cumulative CA&RD returns and cumulative benchmark returns over the period from the end of March 2019 to the end of October 2021. In periods ⑤ (parallel rally) and ⑥ (modest steepening sell-off), both cumulative CA&RD returns, and cumulative benchmark returns decrease after increase, but there is little divergence between the two. In period ⑦ (steepening rally), both returns increase, but the cumulative CA&RD returns greatly exceed the cumulative benchmark returns. The following period ⑧ (modest steepening sell-off), both returns decrease moderately, but cumulative CA&RD returns further exceed cumulative benchmark returns. However, as entering the period ⑨ (intense steepening sell-off), cumulative CA&RD returns are behind the cumulative benchmark returns as both returns decrease significantly. In the period ⑩ (flattening rally), cumulative CA&RD returns exceed cumulative benchmark returns as both returns increase. In the final period ⑪ (flattening sell-off), the cumulative CA&RD returns are far behind the cumulative benchmark returns while both returns decrease. Over the analysis period, the excess return (α) is only 0.19% per annum and it seems that very few excess returns remain considering the transaction cost.

Figure 2: Comparison of cumulative CA&RD returns and cumulative benchmark returns



(Source: Prepared by the authors from FactSet data)

As we examined the optimal portfolio using carry and roll-down strategy during the performance analysis period, with a few exceptions, the portfolio mainly consisted of 6-year or 7-year bond, which accounts for about 80% to 90% of the portfolio, and 20-year or 30-year bond, which is the rest of the portfolio as duration adjustments. This is consistent with Yamada (2000), which assumes that the optimal portfolio of carry and roll-down strategy for the Japanese government bond market is "bullet-type except for a just brief period of time." However, a major difference from Yamada (2000) is that carry and roll-down strategy which is optimally weighted generates only a little profit in the efficient U.S. Treasury market. In order to effectively manage carry and roll-down strategy in the U.S. Treasury market, it is indispensable to have a mechanism to properly switch between "bullet-type" and "barbell-type"

3. Application of factor analysis to U.S. Treasury yields

3.1 Three-Factor model

Litterman and Scheinkman [1991] decomposes yield dynamics into three factors by applying factor analyses to dynamics in U.S. zero-coupon government bond yields. To analyze richness and cheapness of bonds among buckets, we adopted a model where the yield dynamics of the 13 buckets are represented by the dynamics of the three major factors attained from factor analysis to the constant maturity par yields (rather than changes in yields) of the 13 buckets in the U.S. Treasury market shown in Section 2.2, and the residual is defined as an error term. We determine the fair yield is the one expressed by the linear combination of the three major factors, and recognize that the bucket is cheapness if error term is positive and richness if error term is negative.

$$Y_i(t) = a_{i,1}f_1(t) + a_{i,2}f_2(t) + a_{i,3}f_3(t) + \varepsilon_i(t) \qquad (i = 1, \cdots, 13) \qquad (6)$$

Here, each $Y_i(t), \varepsilon_i(t), f_1(t), f_2(t), f_3(t)$ represent the value of the constant maturity par yield of the i bucket, the error term of the i bucket, and the value of the first factor, the second factor, and the third factor at time t (as of the end of each month from the end of April 2017 to the end of March 2019), respectively. $a_{i,1}, a_{i,2,} a_{i,3}$ are defined as the value of the first factor loading, the second factor loading, and the third factor loading of the i bucket.

(Remark)

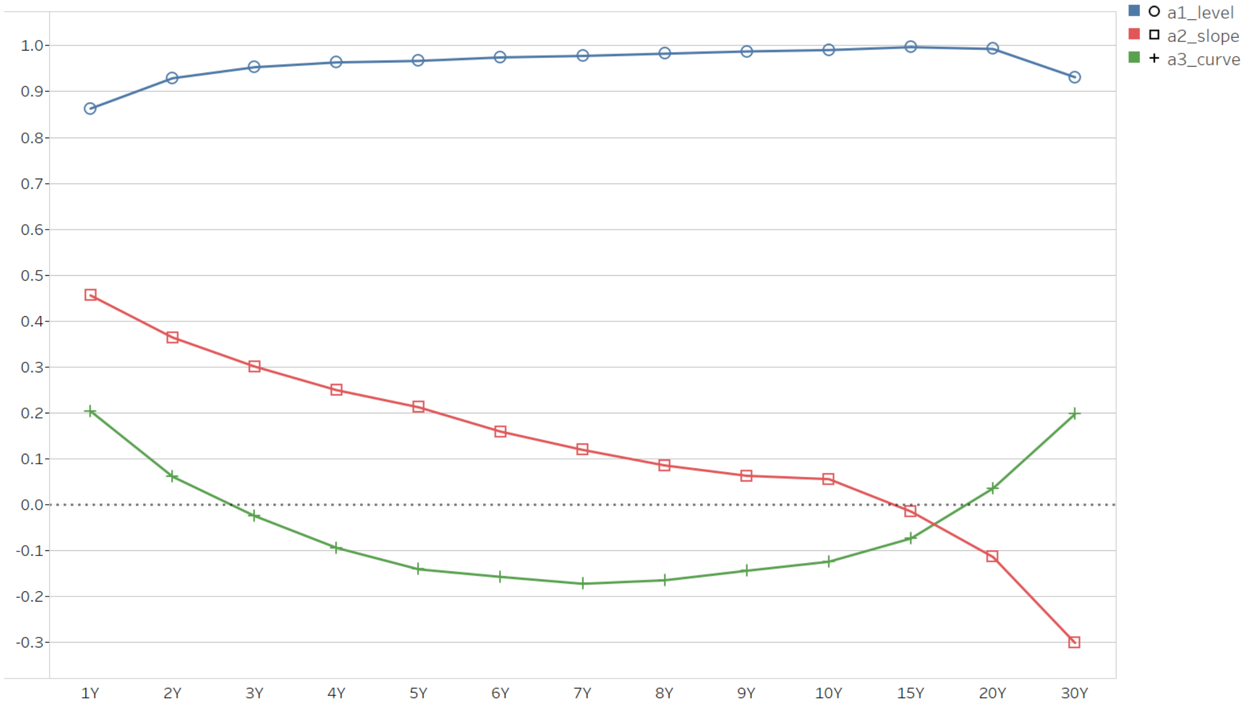
In practice, factor analysis is applied to the standardized yields in equation (6), and an inverse change in standardization is applied to the aggregated three main factors expressed in equation (6) to obtain an appropriate level of yield. The concept of factor analysis is described in detail in Noguchi (2018), and Python was used to implement the model.

3.2 Analysis of U.S. Treasury yields using three-factor model

Figure 3 and Figure 4 show the factor loading and the factors, respectively, which are obtained by applying factor analysis to constant maturity par yields in 13 buckets of the U.S. Treasury market for the period from the end of April 2017 to the end of March 2019. Figure 3 shows the shape of factor loading for the maturity of a Treasury bond. Since the factor loading of the first factor is the same level in various maturity buckets, it could be interpreted as a "level" factor that causes a parallel shift. The second factor as a “slope” factor causes a flattening sell-off or steepening rally (it makes the twist after 15-year), since the factor loading of the second factor becomes smaller as the maturity becomes longer and the sign changes from positive to negative after 15-year. Since the factor loading of the third factor is downward convex mainly in the 6-8-year buckets, when the third factor decreases to negative (or increases to positive) , an upward (or downward) “curvature” appears and decreases (or increases) the performances of each bucket.

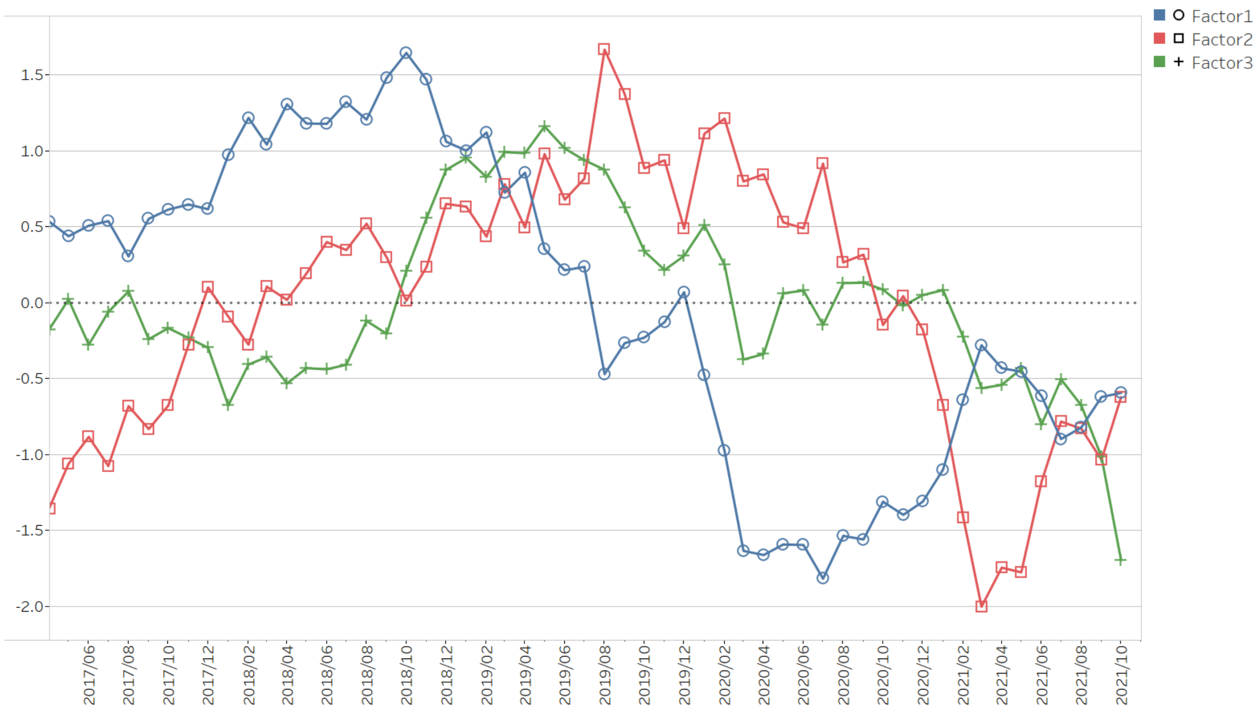
The results are mostly consistent with the those of Litterman and Scheinkman [1991] which apply the factor analysis to yield dynamics. The term-structure model of interest rates in Nelson and Siegel [1987] also utilizes a functional form that assumes these three factors.

Figure 3: Factor Loading



(Source: Prepared by the authors from FactSet data)

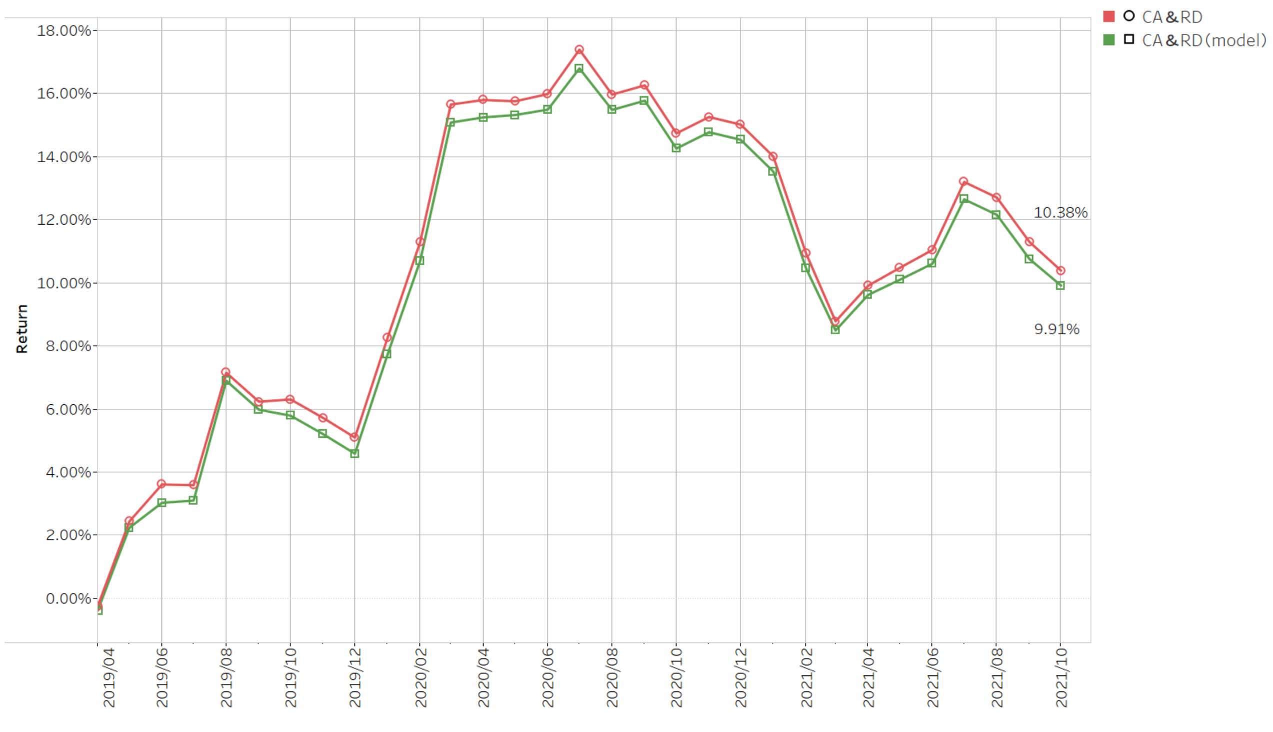
Figure 4: Factors



(Source: Prepared by the authors from FactSet data)

Next, we review the dynamics of each factor in Figure 4. It shows that the first factor dynamics roughly captures the yield dynamics for each U.S. Treasury bond maturity in Figure 1 previously shown. The rise of the second factor flattens the yield curve, as confirmed in Figure 1, for example, in the period from the end of April 2017 to the end of August 2018. The third factor, for example, has decreased since March 2021 and has increased yields, mainly in the 6-8-year buckets. In this course, as seen in Section 2.3, the 6-8-year buckets are underperformed, and the excess return of carry and roll-down strategy accumulated thus far is mostly disrupted. Since the factor model can provide a detailed and accurate representation of yield dynamics for each U.S. Treasury maturity, it seems to have a granularity that can even express the excess return of carry roll-down strategy. Actually, Figure 5 shows both the cumulative return from carry and roll-down strategy based on the fair U.S. Treasury bond yields suggested by the three factor model (excluding the error term on the right-hand side of equation (6) and the cumulative return from carry and roll-down strategy using the actual yields on U.S. Treasury bonds shown in Section 2.3, and it shows that the dynamics of them are almost the same, although the latter exceeds by a little less than 50BP the former. This means that the optimal portfolios applied in carry roll-down strategy for both yields are almost identical. We carefully consider the reasons as follows.

Figure 5: Comparison of cumulative CA&RD returns with cumulative CA&RD (model) returns

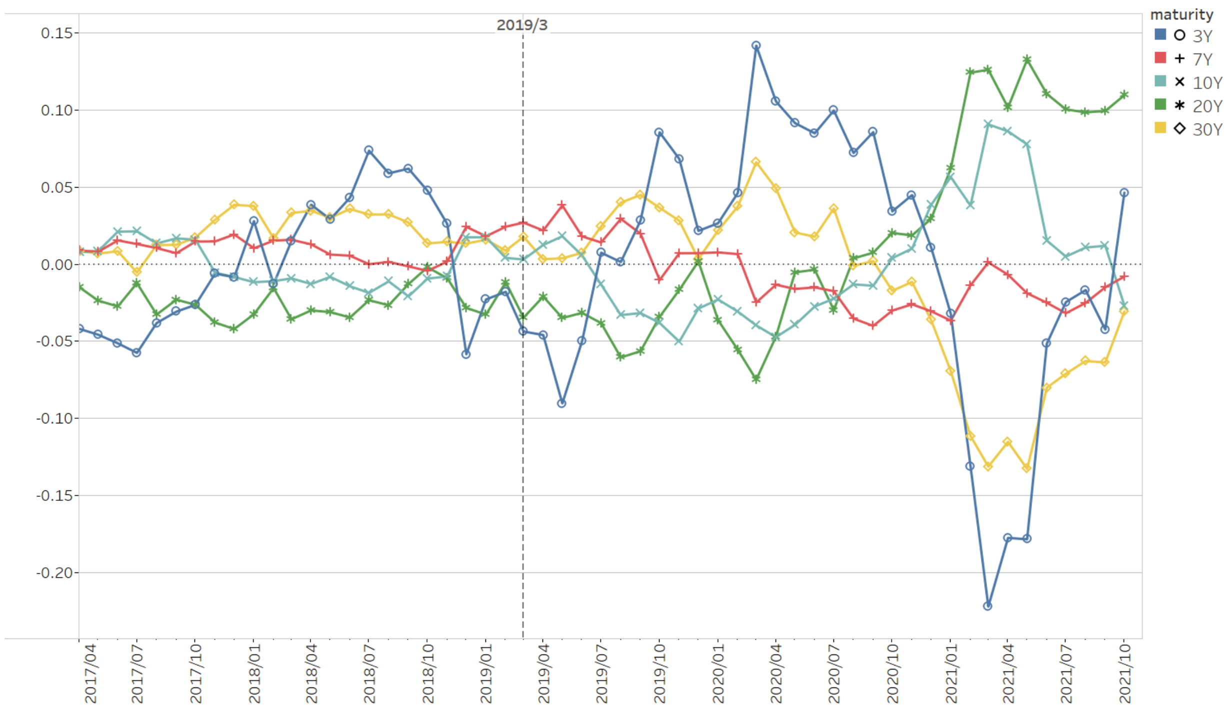


(Source: Prepared by the authors from FactSet data)

If the optimal portfolios are the same, does this indicate that the actual yields on U.S. Treasuries do not get rich or cheap and it would be always mostly the same as the fair yields on U.S. Treasuries based on the three-factor model? Figure 6 shows the error term ($\varepsilon_i(t)$; on the right-hand side of equation (6)) representing richness and cheapness of the U.S. Treasuries with various kinds of maturities for the periods shown in Figure 1. For the in-sample period from the end of April 2017 to the end of March 2019, which is the estimation period for the model, Figure 6 shows the error term just in the factor analysis, and for the out-of-sample period thereafter, the error term is updated by the cross-sectional (across the yield curve) least square regression using the yield curve data at the end of the month and the same factor loading in the in-sample period. Because the factor loading in the in-sample period is also used in the out-of-sample period, the error terms in the out-of-sample period are larger than those in the in-sample period.

Figure 6 shows that during the out-of-sample period in which carry and roll-down strategy is activated, richness and cheapness among maturity buckets appear appropriately as the sign of the error term takes both plus and minus, there were periods when each bucket becomes rich and cheap.

Figure 6: Changes in the error terms (in percentage) between the actual yields of each U.S. Treasury and the yields of the model



(Source: Prepared by the authors from FactSet data)

However, there are many cases where the signs of the error terms are the same between adjacent buckets, for example, in the case of a roll-down from the actual yield of an cheap (rich) bucket to the yield of an cheap (rich) bucket, there is not much difference from the roll-down from the fair yield to the fair yield based on the three-factor model, and the effects of the error terms on carry could be literally within an error interval. In other words, when the return of carry and roll-down derived from the fair yield curve under the yield curve unchanged scenario is simply adopted in determining the optimal portfolio for carry and roll-down strategy, then the optimal portfolio would be determined in a manner that does not reflect the richness and cheapness of each maturity bucket, and in most periods, it would be the bullet-type centered on 6–7-year buckets. By adopting expected returns reflecting richness and cheapness in each bucket, we explore the possibility of the improved carry and roll-down strategy in which optimal portfolios switch appropriately to barbell-type at some point of time.

4. Effective Application of Factor Analysis to carry and roll-down strategies

4.1 Effective application of the three-factor model to carry and roll-down strategies

To start with this Chapter, we revisit the scenario returns for the two kinds of carry and roll-down strategies in Section 3.2, taking 5-year bond as an example.

(i.) The scenario return using the actual yields for each maturity of U.S. Treasury Bonds
(Equation (3): Please refer to above)

$$CA\&RD_{5\to4.5} \approx \frac{1}{P_{0,5}(Y_{0,5})}\left[\frac{C_5}{2} + P'_{0.5,4.5}(Y_{0,5})(Y_{0,4.5} - Y_{0,5}) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(Y_{0,4.5} - Y_{0,5})^2\right] \quad (3)$$

(ii.) The scenario return using fair yields for each maturity of U.S. Treasury Bonds based on the three-factor model
(Excluding the error term on the right side of equation (6))

$$CA\&RD_{5\to4.5} \approx \frac{1}{P_{0,5}(Y_{0,5})}\left[\frac{C_5}{2} + P'_{0.5,4.5}(Y_{0,5})\left(\sum_{j=1}^{3} a_{4.5,j}f_j(0) - \sum_{j=1}^{3} a_{5,j}f_j(0)\right) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})\left(\sum_{j=1}^{3} a_{4.5,j}f_j(0) - \sum_{j=1}^{3} a_{5,j}f_j(0)\right)^2\right] \quad (7)$$

Next, as an effective application of the three-factor model to carry and roll-down strategy, we would like to propose a scenario return that the error term contributes to the return of the roll-down in a way that a certain portion $\gamma(0 < \gamma \le 1)$ of the error term $\varepsilon_5(0)$ which is representing richness /cheapness, that is $\gamma \cdot \varepsilon_5(0)$ is reverting back to 0 over a half-year period.

(iii.) The scenario return based on the proposed method

$$CA\&RD_{5\to4.5} \approx \frac{1}{P_{0,5}(Y_{0,5})}\left[\frac{C_5}{2} + P'_{0.5,4.5}(Y_{0,5})\left(\sum_{j=1}^{3} a_{4.5,j}f_j(0) - \left(\sum_{j=1}^{3} a_{5,j}f_j(0) + \gamma\cdot\varepsilon_i(0)\right)\right) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})\left(\sum_{j=1}^{3} a_{4.5,j}f_j(0) - \left(\sum_{j=1}^{3} a_{5,j}f_j(0) + \gamma\cdot\varepsilon_i(0)\right)\right)^2\right] \quad (8)$$

(Remark)
In the expansion of equation (8), it is assumed that the effect of the third term (static convexity) on the return is small enough to be neglected, so no error term is added to the third term here to simplify the way of calculation.

4.2 Performance analysis on the proposed carry and roll-down strategy

For the period from the end of March 2019 to the end of October 2021, Figure 7 shows the cumulative excess returns from the investment to U.S. Treasury bond index, with the optimal portfolio based on the three scenarios shown in Section 4.1 (i.) through (iii.) being updated monthly. In Figure7, the cumulative excess returns of carry and roll-down strategy based on scenario returns of (i.), (ii.), and (iii.) correspond to (1) CA&RD, (2) CA&RD (model), and (3) CA&RD+α (model), respectively. The difference between (1) CA&RD and (2) CA&RD (model) in Figure 7 is the same as that between (1) CA&RD and (2) CA&RD (model) in Figure 5. We mentioned in Section 3.2, that Figure.5 suggested, "(1) CA&RD exceeds (2) CA&RD (model) by a bit less than 50BP, but the dynamics of both are almost the same." However, according to Figure 7, which is more detailed, (1) CA & RD outperforms (2) CA & RD (model) by nearly 50BP in period⑥(modest steepening sell-off), and then repeats some outperforming and underperforming from the subsequent period⑦ to the final period⑪. After all, the cumulative excess return of (1) CA & RD exceeds the cumulative excess return of (2) CA & RD (model) by only less than 50BP.

Figure 7: Cumulative excess returns against the U.S. Treasury Index

(Source: Prepared by the authors from FactSet data)

Next, we compare (3) CA&RD+α(model), the proposed model of this study with (1) CA&RD. In period⑤(parallel rally), period⑥(modest steepening sell-off), and period⑦(steepening rally), the performances of the two are not significantly different, however from period⑧(modest steepening sell-off) to period⑨(intense steepening sell-off), (3) CA&RD+α(model) outperforms (1) CA&RD by nearly 100BP, and by nearly 35BP in period⑪(flattening sell-off), as a result (3) CA&RD+α(model) outperforms (1) CA&RD by nearly 135BP in total.

In comparison with the portfolio returns of the U.S. Treasury Index (BM), Table 1 shows annualized portfolio returns, excess returns, tracking errors (TE), and information ratios (IR) for (1) CA&RD, (2) CA&RD (model), and (3) CA&RD+α (model) over the performance analysis period. Interestingly, (3) CA&RD+α(model), the proposed model of this study, has an excess return of 0.67%, which is more than three times that of (1) CA & RD, and not only has an excellent such excess return, but also has the smallest TE. For this reason, the IR is 1.10, showing a fairly good performance. The underlying reason to this can be considered as the following. By incorporating a certain portion of the error term $\gamma \cdot \varepsilon_i(0)$ representing richness and cheapness into the returns of the roll-down of 6-7-year bonds, the scenario return of the carry and roll-down of 6-7-year bonds is reduced when the error term is negative and the barbell-type portfolio is selected instead of the bullet-type, even when the bullet-type portfolio of 6-7-year bonds is chosen without the effect. Due to the appropriate switch from the bullet-type portfolio to the barbell-type portfolio, the excess returns could be increased even in the period ⑧ (modest steepening sell-off) and period ⑪ (flattening sell-off) .

Table 1: Annualized returns for each portfolio

Port name	Port Return	Excess Return	TE	IR	win-rate (vsBM)
BM	3.71%	-	-	-	-
①CA&RD	3.90%	0.19%	0.78%	0.24	61.29%
②CA&RD(model)	3.73%	0.01%	0.75%	0.02	54.84%
③CA&RD+α (model)	4.39%	0.67%	0.61%	1.10	70.97%

(Source: Prepared by the authors from FactSet data)

4.3 Primary reason why the proposed method generates excess returns

In this section we consider the reasons why the proposed method produces reasonable excess returns. Since the feature of the proposed method is to use a certain portion of the error term $\gamma \cdot \varepsilon_i(0)$ representing richness and cheapness in the model, which is corrected over a half-year period, the error term $\varepsilon_i(t)$ seems to play a key role to generate excess returns. Therefore, we re-examine equation (6).

$$Y_i(t) = a_{i,1}f_1(t) + a_{i,2}f_2(t) + a_{i,3}f_3(t) + \varepsilon_i(t) \qquad (i = 1,\cdots,13) \qquad (6)$$

Since in factor analysis it is theoretically assumed that the fair yield portion composed by the first to third terms of the right-hand side of equation (6) and the error term are independent, we obtain equation (9) by taking the variance of both sides of equation (6).

$$Var\big(Y_i(t)\big) = Var\left(a_{i,1}f_1(t) + a_{i,2}f_2(t) + a_{i,3}f_3(t)\right) + Var\big(\varepsilon_i(t)\big) \quad (i = 1,\cdots,13) \qquad (9)$$

Here, $Var(\cdot)$ express the variance of the random variable in parentheses.

Computing the variances of the time-series data of actual yields, fair yields from the first to third terms, and the error term, we empirically examine whether equation (9) is established or not.

Theoretically, we assume that equation (9) holds, but in practice, there seems to be a correlation between the fair yield and the error term.

$$Var\big(Y_i(t)\big) = Var\left(a_{i,1}f_1(t) + a_{i,2}f_2(t) + a_{i,3}f_3(t)\right) + Var\big(\varepsilon_i(t)\big) + 2Cov\left(\left(a_{i,1}f_1(t) + a_{i,2}f_2(t) + a_{i,3}f_3(t)\right), \big(\varepsilon_i(t)\big)\right) \qquad (i = 1, \cdots, 13) \tag{10}$$

In equation(10), in case that $\mathrm{Cov}\left(\left(a_{i,1}f_1(t) + a_{i,2}f_2(t) + a_{i,3}f_3(t)\right), \big(\varepsilon_i(t)\big)\right) > 0 (i = 1, \cdots, 13)$ holds, the correlation between the fair yield and the error term is positive. While $\mathrm{Cov}\left(\left(a_{i,1}f_1(t) + a_{i,2}f_2(t) + a_{i,3}f_3(t)\right), \big(\varepsilon_i(t)\big)\right) < 0$ holds, the correlation is negative.

Therefore, if the equality does not hold in equation (9) and [>] ([<]) holds, then the fair yield and error term are thought to be positively (negatively) correlated. The inequality [>] means that "the error term becomes larger when the fair yield increases, and the error term becomes smaller when the fair yield decrease." Thus, when [>] holds in some bucket the bucket will likely to underperform in bearish market and to outperform in bullish market due to the error term. Conversely, The inequality [<] means that "the error term becomes smaller when the fair yield increases, and the error term becomes larger when the fair yield decreases." This suggests that the sector will likely to outperform in bearish market and to underperform in bullish market due to the error term.

Here, Table 2 shows the ratio obtained by dividing the sum of the variances on the right-hand side of equation (9) by the variance on the left-hand side of equation (9). The value in Table 2 shows "negative correlation" if it is larger than 100%, or "positive correlation" if it is less than 100%. In Table 2 for the 1-3- and 15-20-year buckets the indicators are larger than 100% and "negative correlations", and for the 4–10-year buckets the indicators are 100% or less and "positive correlations". In other words, by considering the error term in the proposed method, a barbell-type portfolio consisting of buckets of 1-3 years and 15-20 years was selected for the period ⑧ (modest steepening sell-off) and the period ⑪ (flattening sell-off), and this selection of the portfolio has led to higher excess returns.

Table 2: Variance of actual yields, fair yields, and error terms

		1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	15Y	20Y	30Y
A	Var of actual yields	0.9400	0.9382	0.8988	0.8507	0.7915	0.7419	0.6878	0.6722	0.6427	0.6080	0.5321	0.4125	0.3876
B	Var of fair yields(model)	0.9597	0.9500	0.8951	0.8318	0.7629	0.7174	0.6654	0.6566	0.6356	0.6064	0.5424	0.4275	0.3795
	Var of error terms	0.0002	0.0044	0.0052	0.0028	0.0010	0.0008	0.0004	0.0004	0.0005	0.0009	0.0013	0.0030	0.0022
C	Var of fair yields + Var of error terms	0.9599	0.9544	0.9003	0.8346	0.7639	0.7181	0.6658	0.6570	0.6361	0.6074	0.5437	0.4304	0.3817
	C/A	102.1%	101.7%	100.2%	98.1%	96.5%	96.8%	96.8%	97.7%	99.0%	99.9%	102.2%	104.4%	98.5%

(Source: Prepared by the authors from FactSet data)

5. Summary and future issues

In this study, we first conduct an empirical analysis to what extent excess returns, tracking errors, and information ratios can be obtained when carry and roll-down strategy is adopted in the U.S. Treasury market relative to the U.S. Treasury index, and found that, unlike the Japanese government bond market where the yield curve seems to be controlled by the Bank of Japan's monetary policy, a simple carry and roll strategy is not always an effective strategy in the U.S. Treasury market, which is fairly priced with scarce arbitrage opportunity. Based on the results of this empirical analysis, we considered that in order for carry and roll-down strategy in the US Treasury market to work effectively, a mechanism is necessary for construction of the optimal portfolio to properly switch between "bullet-type" and "barbell-type." To construct this mechanism, we revisited Litterman and Scheinkman [1991] for the first time in 30 years to incorporate the error term obtained by applying factor analysis to the yield itself into the scenario returns of carry and roll-down strategy. Carry and roll-down strategy based on the proposed model was confirmed to have some potential to obtain excess returns possibly even when U.S. Treasuries are bearish.

There are three main issues to be addressed in the future as follows. (I) In this study, we could only show the results of empirical analysis in a relatively short-term period after the end of April 2017, when index weights for each maturity of U.S. Treasuries are easily available for GPIF. Long-term empirical analysis with an analysis period of the past 20 years will be necessary. (II) When conducting the empirical analyses based on the yield data in the past long-term periods, the optimality of the setting itself should be examined. Specifically, answers to the following questions need to be explored. Here the factor loading obtained by applying factor analysis to the monthly yield data for the past two years was fixed and they were used for the performance analysis period, but is the length of the past two years appropriate when estimating the factor loading? When calculating scenario returns, we adopted $\gamma = 1/2$ in the empirical analysis based on the assumption that a certain portion of the error term $\gamma \cdot \varepsilon_i(0)$ representing richness and cheapness would be corrected in six months, but to what extent should it be appropriate to be set? (III) It is necessary to consider the transaction costs required to rebalance the portfolio.

Although there are many challenges to be overcome as described above, we believe that it is possible to utilize the approach of this study to a certain extent in the effective management of carry and roll-down strategy in the U.S. Treasury market, which is fairly priced in a manner that generally meets no arbitrage condition.

Appendix: Derivation of equation (1)

$$P_{0,5}(x) = \frac{C_5/2}{\left(1+\frac{x}{2}\right)^1} + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^2} + \cdots + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^9} + \frac{C_5/2+100}{\left(1+\frac{x}{2}\right)^{10}} \tag{A.1}$$

$$P_{0.5,4.5}(x) = C_5/2 + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^1} + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^2} + \cdots + \frac{C_5/2}{\left(1+\frac{x}{2}\right)^8} + \frac{C_5/2+100}{\left(1+\frac{x}{2}\right)^9} \tag{A.2}$$

The Taylor expansion of equation (A.2) to the second order around the 5-year bond yield $Y_{0,5}$ is as follows.

$$P_{0.5,4.5}\left(\tilde{Y}_{0.5,4.5}\right) \approx P_{0.5,4.5}(Y_{0.5}) + P'_{0.5,4.5}\left(Y_{0,5}\right)\left(\tilde{Y}_{0.5,4.5} - Y_{0,5}\right) + \frac{1}{2}P''_{0.5,4.5}\left(Y_{0,5}\right)\left(\tilde{Y}_{0.5,4.5} - Y_{0,5}\right)^2 \tag{A.2'}$$

The change in bond prices over half a year from the present (the numerator on the left side of equation (1)) is attained by substituting (A.1) and (A.2'),

$$P_{0.5,4.5}\left(\tilde{Y}_{0.5,4.5}\right) - P_{0,5}\left(Y_{0,5}\right) \approx \left(P_{0.5,4.5}(Y_{0.5}) - P_{0,5}\left(Y_{0,5}\right)\right) + P'_{0.5,4.5}\left(Y_{0,5}\right)\left(\tilde{Y}_{0.5,4.5} - Y_{0,5}\right) +$$

$$\frac{1}{2}P''_{0.5,4.5}\left(Y_{0,5}\right)\left(\tilde{Y}_{0.5,4.5} - Y_{0,5}\right)^2. \tag{A.3}$$

And expand each of the first to third terms on the right side of equation (A.3),

〔term 1〕 $= \left(P_{0.5,4.5}(Y_{0.5}) - P_{0,5}(Y_{0,5})\right)$

Substituting $Y_{0,5}$ into (x) in equations (A.1) and (A.2) to organize them,

$$= \left(C_5/2 + \frac{C_5/2}{\left(1+\frac{Y_{0,5}}{2}\right)^1} + \frac{C_5/2}{\left(1+\frac{Y_{0,5}}{2}\right)^2} + \cdots + \frac{C_5/2}{\left(1+\frac{Y_{0,5}}{2}\right)^8} + \frac{C_5/2+100}{\left(1+\frac{Y_{0,5}}{2}\right)^9}\right) - \left(\frac{C_5/2}{\left(1+\frac{Y_{0,5}}{2}\right)^1} + \frac{C_5/2}{\left(1+\frac{Y_{0,5}x}{2}\right)^2} + \cdots + \frac{C_5/2}{\left(1+\frac{Y_{0,5}}{2}\right)^9} + \frac{C_5/2+100}{\left(1+\frac{Y_{0,5}}{2}\right)^{10}}\right)$$

$$= C_5/2 + \frac{100}{\left(1+\frac{Y_{0,5}}{2}\right)^9} - \frac{C_5/2+100}{\left(1+\frac{Y_{0,5}}{2}\right)^{10}}$$

$$\approx C_5/2 + 100\left(1 - 9\cdot\frac{Y_{0,5}}{2}\right) - (C_5/2+100)\left(1 - 10\cdot\frac{Y_{0,5}}{2}\right)$$

$$= C_5/2 + 100 - 450Y_{0,5} - C_5/2 + \frac{10}{4}C_5\cdot Y_{0,5} - 100 + 1000\cdot\frac{Y_{0,5}}{2}$$

$$= 50\cdot Y_{0,5} + \frac{5}{2}C_5\cdot Y_{0,5}.$$

And, it is considered that $C_5\cdot Y_{0,5}$ is sufficiently small and negligible, and considering the assumption ($C_5 = Y_{0,5}\cdot 100$) of Per bond, the following could be obtained.

$$\approx 100\cdot\frac{Y_{0,5}}{2} = C_5/2$$

〔term 2〕 $= P'_{0.5,4.5}(Y_{0,5})(\tilde{Y}_{0.5,4.5} - Y_{0,5})$

$$= P'_{0.5,4.5}(Y_{0,5})(\tilde{Y}_{0.5,4.5} - Y_{0,4.5} + Y_{0,4.5} - Y_{0,5})$$

〔term 3〕 $= \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(\tilde{Y}_{0.5,4.5} - Y_{0,5})^2$

$$= \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(\tilde{Y}_{0.5,4.5} - Y_{0,4.5} + Y_{0,4.5} - Y_{0,5})^2$$

$$= \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})\left((\tilde{Y}_{0.5,4.5} - Y_{0,4.5})^2 + 2(\tilde{Y}_{0.5,4.5} - Y_{0,4.5})(Y_{0,4.5} - Y_{0,5}) + (Y_{0,4.5} - Y_{0,5})^2\right)$$

$$\approx \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})\left((\tilde{Y}_{0.5,4.5} - Y_{0,4.5})^2 + (Y_{0,4.5} - Y_{0,5})^2\right)$$

In the final approximation, $2(\tilde{Y}_{0.5,4.5} - Y_{0,4.5})(Y_{0,4.5} - Y_{0,5})$ is assumed to be small enough.

Substituting [term1 to term3] into equation (A.3), the price change (numerator on the left side of equation (1)) for half a year from the present is as follows.

$$\left[\frac{C_5}{2} + P'_{0.5,4.5}(Y_{0,5})(Y_{0,4.5} - Y_{0,5}) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(Y_{0,4.5} - Y_{0,5})^2 + P'_{0.5,4.5}(Y_{0,5})(\tilde{Y}_{0.5,4.5} - Y_{0,4.5}) + \frac{1}{2}P''_{0.5,4.5}(Y_{0,5})(\tilde{Y}_{0.5,4.5} - Y_{0,4.5})^2\right]$$

Therefore, the right side of equation (1) is obtained.

References

Cochrane, J. H. and M. Piazzesi [2005] “Bond Risk Premia,” The American Economic Review 95 (1), pp. 138-160.

Koijen, R. S. J., T. J. Moskowitz, L. H. Pedersen, and E. B. Vrugt [2015]” Carry,” Working Paper.

Litterman, R. and J. Scheinkman [1991] Common Factors Affecting Bond Returns, The Journal of Fixed Income, Vol. 1, No. 1, pp. 54-61.

Nelson, C. R. and A. F. Siegel [1987] Parsimonious Modeling of Yield Curves, Journal of Business, Vol. 60, No. 4, pp. 473-489.

H. Reisman and G. Zohar [2004a] “Instantaneous Mean-Variance Analysis of Bond Returns,” The Journal of Fixed Income, Vol. 14, No. 1, pp.32-39.

H. Reisman and G. Zohar [2004b] “Short-Term Predictability of the Term Structure,” The Journal of Fixed Income, Vol. 14, No. 3, pp.7-14.

H. Reisman and G. Zohar [2004c] “Excess yields in bond hedging,” Risk, Vol. 17, No. 12, pp.93-97.

Tadashi Kikugawa, Tomonori Uchiyama, Mamoru Motohiro, Shou Nishiuchi [2017] "Bond Fund Performance and a Smart Beta Strategy in Japan," "Securities Analysts Journal" 55 (2), pp. 69-80. *(published in Japanese)*

Genzo Kimura [2021] "What Makes Interest Rates So Low in Japan, Euro Area, and the US? " Securities Analysts Journal, 59 (11), pp. 77-85. *(published in Japanese)*

Toshio Sakudo [2010] "Risk premium of Bonds," "Securities Analysts Journal" 48 (8), pp. 5-13. *(published in Japanese)*

Edition by the Securities Analysts Association of Japan/Supervised by Yukihiro Asano and Shigeki Sakakibara/Keisuke Ito, Seiji Ogishima and Takashi Suwabe (2020), "New - Securities Investment Theory II: Practical Edition," Nikkei Business Publications, Inc. *(published in Japanese)*

Hiroshi Noguchi [2018] "Introduction to Multivariate Analysis Learned from Illustrations and Numerical Examples", The Japanese Standards Association Group *(published in Japanese)*

Masashi Hayashida [2005] "Risks and Returns of Bonds," Securities Analysts Journal, 43 (9), pp. 18-28. *(published in Japanese)*

Hirofumi Hoshika, Koichi Miyazaki [2007] "Investment Strategy for Japanese Government Bonds Based on a Two-Factor Model" "Operations Research" 52, (2) pp. 108-114. *(published in Japanese)*

Satoshi Yamada [2000] "Empirical Study of Risk Premiums in the JGB Market and Its Application to Investment Strategies," Securities Analysts Journal, 38 (12), pp. 32-62. *(published in Japanese)*