GPIF working paper

Japanese Private Real Estate Models and Portfolio Selection

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EXECUTIVE SUMMARY

- Private real estate is attracting attention as one of the alternative investment assets from the viewpoint of high income gain and its low correlation with traditional assets due to low liquidity. The prices of privately placed real estate are mainly based on "appraisal prices" by real estate appraisers. The return of private real estate has autocorrelation due to the smoothing effects of appraisals, and therefore, appropriate de-smoothing of the return is necessary for practical use.
- However, even though it is de-smoothed, the return is still based only on appraisal prices, so it cannot be said to be based on prices actually traded in the market (hereinafter referred to as "transaction-based prices").
- To solve the issue, we propose a modeling of mean and variance of the transaction-based return for privately placed real estate, using both the de-smoothed returns of privately placed real estate and the J-REIT returns obtained from the listed market.

1. Introduction

Although global interest rates are in a phase of rising around the first half of 2022, the bond yields in Japan remains extremely low, and under such circumstances, the risk-return characteristics of J-REITs, which are Japanese version of the listed real estate investment trust, are attracting attention. In addition, privately placed REITs, which are less liquid than J-REIT, are expected to generate even higher returns with an added liquidity risk premium. Against this backdrop, the Japanese real estate securitization market has grown to a market size of 40 trillion yen as of 2022.

Tanabe (2022) evaluates Japanese real estate securitization as one of the few innovations, reviews the history of the real estate securitization market to date, and summarizes the future growth direction of the market with the keywords Arbitrage, Borderless, and Concentration. In particular, with regard to "Arbitrage," the report states that the market for securitized real estate will further promote the integration of real estate and finance, as well as arbitrage not only within the real estate market, but also with other financial products. The securitization market is expected to serve as a platform for connecting the real estate market and the financial market. Among the assets traded in the securitized real estate market, J-REITs, which are listed REITs, have some liquidity, and their prices are marked to market in a daily basis as traditional assets, so it is relatively easy to analyze their relative value to traditional assets. Arbitrage between REITs and real estate has already been pointed out by Kawaguchi (2004), it stated that the ratio of a REIT's stock price (P) to its corresponding NAV provides investors with extremely important information. Kawaguchi (2004) explains that the indicator can be used for arbitrage, citing a study by Gentry et al. (2003) on U.S. REITs. Also, Shimada, Miyazaki, and Oishi (2021) discuss whether alternative assets are attractive or not, in comparison with traditional assets in terms of risk-return characteristics and the effect of including then in a portfolio consisted with traditional assets. As a first step to examine these issues, Shimada, Miyazaki, and Oishi (2021) considers J-REITs as alternative assets and discuss the necessity of managing J-REIT separately as an independent asset class in a portfolio.

Tokushima (2022) points out that among alternative investments, investments in low-liquidity assets such as real estate should be treated separately from traditional assets classes and considered as a new asset class when the investment amount expands, because their risk-return characteristics differ from those of traditional assets. It goes on to say, "Due to the highly idiosyncratic nature of the investments, it is probably inappropriate to use the historical standard deviation and correlation coefficients as they are in the optimization for asset allocation."

In order to conduct arbitrage trade within real estate market, as well as between real estate market and financial markets, it is necessary that risk-return characteristics can be compared with a high degree of accuracy. As mentioned above, the price for private REITs and real estate is based on the appraisal value, and is smoothed so that it does not dynamically change from the past appraisal value. In other words, with regard to privately placed REITs and real estate, it cannot be assumed that daily returns are independently, identically, normally distributed as traditional assets and the simple mean-variance method is inappropriate.

In this study, we first remove the autocorrelation inherent in appraisal price returns for real estate by de-smoothing, and transform the returns so that they are independently, identically, normally distributed. We call them "de-smoothed returns." There are two main methods of de-smoothing according to Spencer, Andrei, and Andrea (2020). One is the Geltner's (1991, 1993) approach that relies on AR(H)-type time series models, mainly for real estate returns, and the other is Getmansky, Lo, and Makarov (2004) that relies on MA(H)-type time series models, mainly for hedge fund and PE returns. In this study, we adopt Geltner's (1991, 1993) approach because the subject is the appraised price return of real estate.

Next, regarding the modeling of mean and variance of real estate return, preceding researches have simply obtained average and volatility of the de-smoothed returns. The problem in it is that the price assumed by real estate appraisers does not necessarily coincide with the transaction price. Therefore, the de-smoothed return itself cannot be regarded as the return based on the transaction price of privately placed real estate. Therefore, focusing on the fact that the J-REIT price is a transaction price with transfer of real estate ownership, we propose a model of mean and variance on a transaction-based return for privately placed real estate by utilizing both the de-smoothed return and the J-REIT return. An empirical analysis will also be conducted to understand the characteristics of the model. The model will make it possible to compare risk-return characteristics among real estate and financial market with a high degree of accuracy.

In the empirical analysis of portfolio selection, the optimal portfolio selection will be examined from the perspective of Sharpe ratio maximization and risk-return by adding low-liquidity private real estate and J-REIT, to the four traditional assets. In doing so, we will utilize data from the Association for Real Estate Securitization (ARES) for privately placed real estate, and will analyze the risk-return of privately placed real estate in three ways: "based on the original ARES data (hereafter, ARES)", "based on the de-smoothed ARES data (hereafter, de-smoothed ARES)," and "based on our proposed model (hereafter, the model)." The usefulness of the model for portfolio selection will be examined.

The paper is organized as follows. Section 2 compactly introduce de-smoothing method that relies on an AR(H)-type time series model for real estate return. Section 3 proposes the model to derive mean and variance of transaction-based return for privately placed real estate and identifies the characteristics of the model through empirical analysis. Section 4 presents an empirical analysis of portfolio selection by adding two assets (privately placed real estate and J-REIT) to four traditional assets from the perspective of Sharpe ratio maximization. The final section is accompanied by a summary and future issues.

2. De-smoothing for real estate returns (Geltner's (1991, 1993) approach)

2.1 De-smoothed ARES Returns

The observable return for any given period (ARES return) R_t^o shall be formed as in equation (1) using the de-smoothed return for this period (the return that appraiser assumes without smoothing) R_t and observable returns prior to this period (R_{t-1}^o, $R_{t-2}^o, \cdots, R_{t-H}^o$).

$$R_t^o = \theta^{(0)} \cdot R_t + \sum_{h=1}^{H} \theta^{(h)} \cdot R_{t-h}^o \tag{1}$$

where each θ is a parameter expressing the degree of obsolescence in the observable returns (the older the data, the less it should be influenced) and satisfy the following conditions.

$$\sum_{h=0}^{H} \theta^{(h)} = 1 \tag{2}$$

In addition, the de-smoothed return at the period t R_t follows the Brownian motion with drift in equation (3).

$$R_t = \mu + \eta_t \quad , \ \mathrm{E}[\eta_t] = 0 \ , \ \ \eta_t \sim i.i.d \tag{3}$$

Substituting equation (3) into equation (1) and rearranging, we obtain equation (4) as follows.

$$\begin{aligned} R_t^o &= \theta^{(0)} \cdot (\mu + \eta_t) + \sum_{h=1}^{H} \theta^{(h)} \cdot R_{t-h}^o \\ &= \theta^{(0)} \cdot \mu + \sum_{h=1}^{H} \theta^{(h)} \cdot R_{t-h}^o + \theta^{(0)} \cdot \eta_t \\ &= \left(1 - \sum_{h=1}^{H} \theta^{(h)}\right) \cdot \mu + \sum_{h=1}^{H} \theta^{(h)} \cdot R_{t-h}^o + \theta^{(0)} \cdot \eta_t \\ &= \mu + \sum_{h=1}^{H} \theta^{(h)} \cdot (R_{t-h}^o - \mu) + \theta^{(0)} \cdot \eta_t \end{aligned} \tag{4}$$

2.2 Parameter estimation for de-smoothed returns

The observable mean-deducted return X_t is defined as equation (5) by deducting the mean from the observable return R_t^o. The term $\theta^{(0)} \cdot \eta_t$ in equation (4) being taken as the error term $\varepsilon_t = \theta^{(0)} \cdot \eta_t$, it can be quantified from the regression equation (6) regarding X_t.

$$X_t = R_t^o - \mu \tag{5}$$

$$X_t = \theta^{(1)} X_{t-1} + \theta^{(2)} X_{t-2} \cdots + \theta^{(H)} X_{t-H} + \varepsilon_t \tag{6}$$

Once the parameters of the multiple regression equation (6), $\theta^{(1)}$, $\theta^{(2)}$, $\cdots$, $\theta^{(H)}$ and the error term ε_t are obtained, $\eta_t = \frac{\varepsilon_t}{1-\sum_{h=1}^{H}\theta^{(h)}}$ is also computed. Substituting it into equation (3), we are able to estimate the de-smoothed return R_t as equation (7).

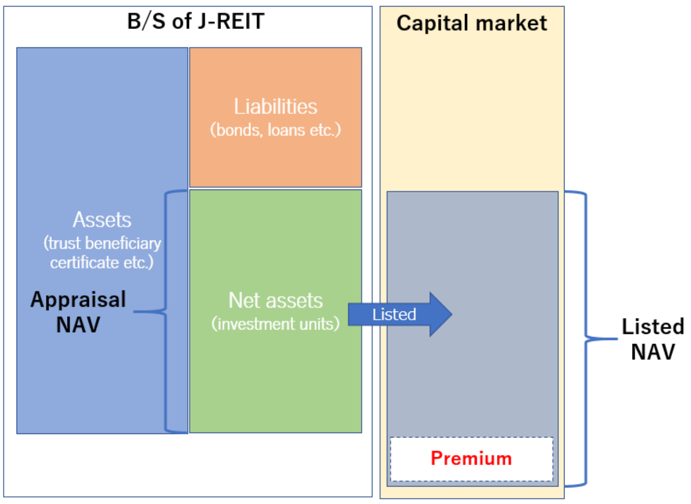
$$R_t = \mu + \frac{\varepsilon_t}{1-\sum_{h=1}^{H}\theta^{(h)}} \tag{7}$$

3. Model and parameter estimation

3.1 Model

In section 2, we first follow preceding research to model the de-smoothed return R_t as equation (3). In constructing our model, we were conscious of the following question: "Since the price assumed by real estate appraisers does not necessarily coincide with the transaction price, can we not regard the de-smoothed return as a return that represents the actual investment return?" More specifically, the situation is explained by using the two NAVs shown in Figure 1. The left side of the balance sheet (the Assets) of J-REIT has a lot of properties which are based on the appraised price real estate appraisers assign. We call the part of the Assets minus the liabilities (bonds, loans, etc.) "appraisal NAV". The price of the investment units (which is based on the appraised price) corresponds to the original ARES price data. Although the appraisal value of individual properties is basically updated twice a year, this does not mean that the properties can be traded at the price. In contrast, if the investment units are listed and traded in the capital market on a daily basis, the investment units (hereinafter, referred to as "listed NAV") are marked to market on a daily basis. Even if there is a divergence between the two, they should ultimately eventually converge because they are the same real estate.

【Figure 1: Appraisal NAV and Listed NAV】



(Source: Prepared by the authors based on data from Japan Real Estate Institute)

For this reason, in order to estimate the transaction-based return of real estate traded as private placements, de-smoothed return R_t alone is not sufficient and the J-REIT return should also be utilized. Therefore, in our model, we consider the situation where the transaction price deviates from the price assumed by the real estate appraiser, due in part to the effects of transaction costs (equivalent to the Bid-Offer Spread in liquid assets) and other factors. The mean (μ) and variance (σ^2) of the transaction-based return (R_t^P) are estimated using both de-smoothed return R_t and the J-REIT return. Utilizing them, the transaction-based return (R_t^P) is modeled by transforming the de-smoothed return R_t.

Where, the de-smoothed return R_t is newly denoted R_t^A. Equation (7) becomes equation (8) in the new notation.

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$$R_t^A = \mu_A + \varepsilon_t^A, \quad \varepsilon_t^A \sim i.i.d.N\,(0,\ (\sigma^A)^2) \tag{8}$$

In addition, the J-REIT return R_t^J follows equation (9) and equation (10).

$$R_t^J = \mu_J + \varepsilon_t^J, \quad \varepsilon_t^J \sim i.i.d.N\,(0,\ (\sigma^J)^2) \tag{9}$$

$$correlation(R_t^J, R_t^A) = \rho_{J,A} \tag{10}$$

The transaction-based return on real estate traded as a private placement R_t^P is modeled as follows.

$$\frac{R_t^P - \mu}{\sigma} := \frac{R_t^A - \mu_A}{\sigma^A}. \tag{11}$$

Rewriting equation (11),

$$R_t^P := \left(\mu - \frac{\sigma}{\sigma_A}\mu_A\right) + \frac{\sigma}{\sigma_A}R_t^A. \tag{12}$$

Where,

$$\mu = \frac{1}{2}(\mu_A + \mu_J) \tag{13}$$

$$\sigma^2 = \frac{1}{2}((\sigma^A)^2 + (\sigma^J)^2) + \frac{1}{4}\left(\mu_A - \mu_J\right)^2 \tag{14}$$

$$\rho = \left[\rho_{J,A}\sigma^A\sigma^J - \frac{1}{4}\left(\mu_A - \mu_J\right)^2\right] \Big/ \left[\frac{1}{2}((\sigma^A)^2 + (\sigma^J)^2) + \frac{1}{4}\left(\mu_A - \mu_J\right)^2\right] \tag{15}$$

With the background given in this section, we now provide the way of estimating the parameters of the model in equation (13) through (15). R_t^A and R_t^J are assumed to be given by equation (8) through equation (10) follow the two-dimensional normal distributions in equations (16-1) and (16-2).

$$f(R_t^A, R_t^J, \boldsymbol{\theta}) = \frac{1}{2\pi\sigma^A\sigma^J\sqrt{1-(\rho_{J,A})^2}} exp\left\{-\frac{1}{2\left(1-(\rho_{J,A})^2\right)} Q\left(R_t^A, R_t^J\right)\right\} \tag{16-1}$$

$$Q(x_i, y_i) = \frac{\left(R_t^A - \mu_A\right)^2}{(\sigma^A)^2} - 2\rho_{J,A}\frac{(R_t^A - \mu_A)\left(R_t^J - \mu_J\right)}{\sigma^A\sigma^J} + \frac{\left(R_t^J - \mu_J\right)^2}{(\sigma^J)^2} \tag{16-2}$$

As noted earlier, total capitalization units and market capitalization units should ultimately converge on the same risk-return characteristics, even if there is a temporary divergence between the two, since they are both valuations of the same property in a private placement and in the capital markets. Therefore, the average μ and the variance σ^2 of the transaction price return in the private placement R_t^P are modeled with the restriction of parameters as $\mu_A = \mu_J = \mu$, $\sigma^A = \sigma^J = \sigma$, and $\rho_{J,A} = \rho$ in equations (16-1) and (16-2) and estimated by way of maximum likelihood method. See the Appendix for details on the derivation of equation (13) through (15).

(Remark)
This section presents a method for estimating the transaction-based return in a private placement for the total equity units of the entire real estate in which the J-REIT invests. To estimate the transaction-based return in the case of a private placement of individual properties, it is necessary to construct data on the listed NAV (for J-REIT return) customized to match the region and use of the property, in addition to the appraisal NAV (for ARES return) data for the property in question.

3.2 Parameter estimation

3.2.1 Data

The data in the parameter estimation is as follows.

(1) Time series of the de-smoothed return R_t^A obtained by de-smoothing the return of the ARES Japan Property Index (hereinafter referred to as the ARES return) for the equity portion of the individual properties held by J-REITs.

(2) Time series data for J-REIT return R_t^J.

3.2.2 Estimation procedure

The procedure for parameter estimation for the model is as follows.

Step1：

Computing the mean and the variance of data (1), we obtain the mean μ_A and the variance $(\sigma^A)^2$, respectively. As the same manner, computing the mean and variance of data (2), we obtain the mean μ_J and the variance $(\sigma^J)^2$, respectively. In addition, computing the correlation coefficient for data (1) and data (2), we obtain the correlation coefficient $\rho_{J,A}$.

Step2：

The estimated parameters in Step 1 are substituted into equation (13) through (15) to obtain the parameter of the model μ, σ, and ρ.

3.3 Empirical analysis to capture the characteristics of the model

3.3.1 Parameter estimation for de-smoothed returns

First, the autoregressive parameters obtained by applying the AR(H) model to the monthly ARES returns are estimated by the maximum likelihood method; the number of lags H in the AR model is selected based on the AIC criterion. The return obtained by substituting the parameter values estimated here into equation (7) is the de-smoothed ARES return. Next, to confirm that these de-smoothed returns are not autocorrelated, the AR(H) model is applied again to the de-smoothed returns to check the significance of the estimated autoregressive parameters, and if the null hypothesis that the autoregressive parameters are zero is rejected, the returns are considered to be autocorrelated. The estimated parameters obtained using monthly ARES returns from April 2003 to December 2021 are shown in Table 1. The results obtained by applying the AR (10) model to the monthly ARES returns show that the value of the autoregressive coefficient for lag 1 is positive and its t-value of 18.62 is significant, suggesting the existence of a strong positive autocorrelation.

Furthermore, the autoregressive parameters for lags 5 and 10 are negative and their absolute t-values are significant above 3, suggesting the necessity of taking the number of lags up to 10 to eliminate autocorrelation. In contrast, the results obtained by applying the AR (10) model to the de-smoothed returns show that the t-values of all the autoregressive parameters are less than about 1, confirming that the de-smoothed returns were indeed obtained by properly removing the autocorrelation inherent in the monthly ARES returns.

【Table 1 ：Autoregressive coefficients and t-values】

ARES returns

	μ	$\theta^{(1)}$	$\theta^{(2)}$	$\theta^{(3)}$	$\theta^{(4)}$	$\theta^{(5)}$	$\theta^{(6)}$	$\theta^{(7)}$	$\theta^{(8)}$	$\theta^{(9)}$	$\theta^{(10)}$
coefficients	0.00	1.25	0.16	-0.15	-0.07	-0.43	0.06	0.02	0.14	0.21	-0.22
t-values	2.17	18.62	1.48	-1.35	-0.67	-4.03	0.57	0.23	1.37	2.02	-3.29

de-smoothed returns

	μ	$\theta^{(1)}$	$\theta^{(2)}$	$\theta^{(3)}$	$\theta^{(4)}$	$\theta^{(5)}$	$\theta^{(6)}$	$\theta^{(7)}$	$\theta^{(8)}$	$\theta^{(9)}$	$\theta^{(10)}$
coefficients	0.01	-0.03	0.03	0.04	0.02	-0.01	-0.06	-0.03	0.02	0.05	0.09
t-values	2.36	-0.40	0.37	0.54	0.30	-0.15	-0.84	-0.45	0.34	0.67	1.23

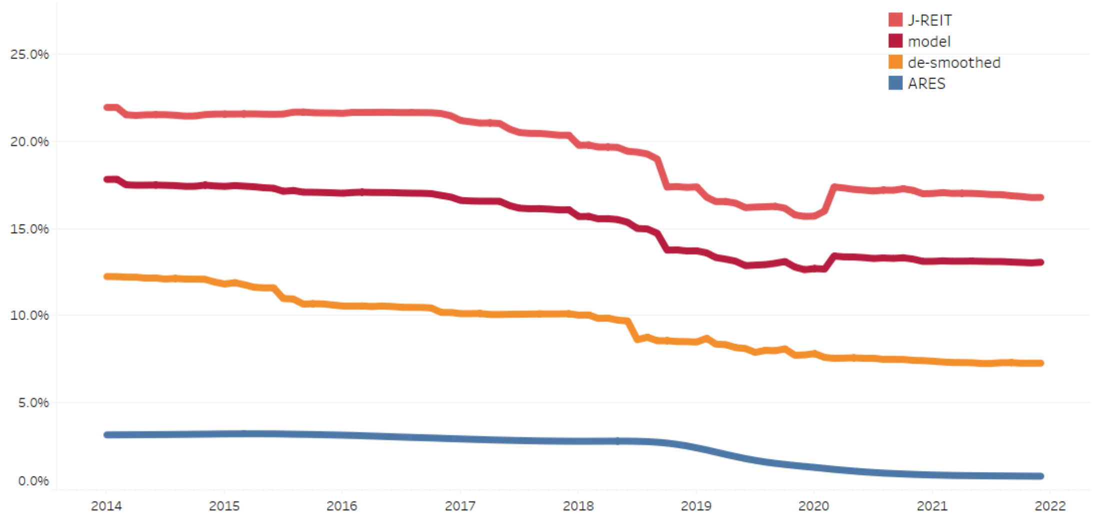
(Source：Authors' compilation from ARES data)

3.3.2 Volatility and Correlation Coefficients

To get an idea of the level of volatility generated by our model, the volatility of the model, the volatility of the ARES returns, the volatility of the de-smoothed returns, and the volatility of the J-REIT returns, four in total, are obtained based on monthly returns for the 10-year and three-year periods and compared on a rolling basis. With respect to the correlation coefficients, because the model revamps only the volatility of the de-smoothed return to make it more realistic by taking market return into account, the correlation coefficient between the transaction-based return traded in private placement and other assets' returns is still assumed to be equal to that between the de-smoothed return and other assets returns. We compare correlation coefficients computed based on monthly returns for two periods, the past 10 years and the past three years on a monthly basis among the correlation coefficient between the ARES return and the composite benchmark return (representing the return of a portfolio with one-quarter each of domestic bonds, domestic stocks, foreign bonds, and foreign stock; the same applies hereafter), the correlation coefficient between the de-smoothed return and the composite benchmark return, and the correlation coefficient between the J-REIT return and the composite benchmark return. These correlation coefficients will be an important factor when discussing the Sharpe ratio for a portfolio that incorporates real estate into a portfolio of four traditional assets.

Long term volatilities (obtained from rolling over the past 10 years) are shown in Figure 2. The four long-term volatilities have generally the same time-series trends but differ significantly in level. A closer look shows that for "J-REIT" and "the model," volatility, which was high in the 2014-2017 period, declined rapidly in the period from 2017 to 2019, rose again in 2020, and has remained stable since then. This is due to several factors such as economic shock in the past 10 years, for example, the GFC affecting the volatility from 2014 to 2017 and the COVID-19 Shock influencing on the volatility around 2020. Interestingly, with respect to the “de-smoothed”, the volatility, which was high during the 2014 period, has decreased prior to that of “J-REIT” and “the model” and was not affected by the COVID-19 shock. As for “ARES,” the autocorrelation of monthly return is high, so it can be noticed that the time when the impact of the GFC on the volatility is clearly slipping away is about one to two years later than that of “J-REIT” and “the model,” and it has not been affected by the COVID-19 Shock.

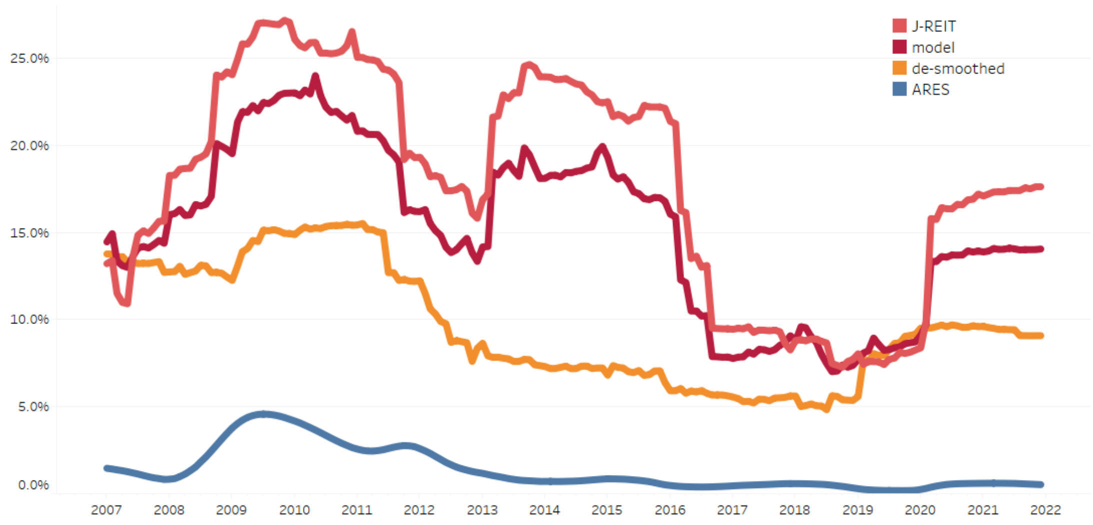
【Figure 2 : Long-term volatility (120-month rolling, annualized)】



(Source: Author’s calculations based on FactSet and ARES data)

Short-term volatility (obtained on a rolling basis based on the last three years of data) is shown in Figure 3. As expected, it can be seen that the change in short-term volatility is much larger than that in long-term volatility. It is interesting to note that the increase in short-term volatility is also confirmed in “de-smoothed ARES” and “ARES” due to the impact of the GFC, and in particular, for “de-smoothed ARES”, it is also affected by the COVID-19 shock to some extent. In addition, the level of short-term volatilities of “J-REIT,” “the model,” and “de-smoothed ARES” were almost the same when they were not affected by the economic shocks such as the GFC in 2008 and the COVID-19 Shock in 2020.

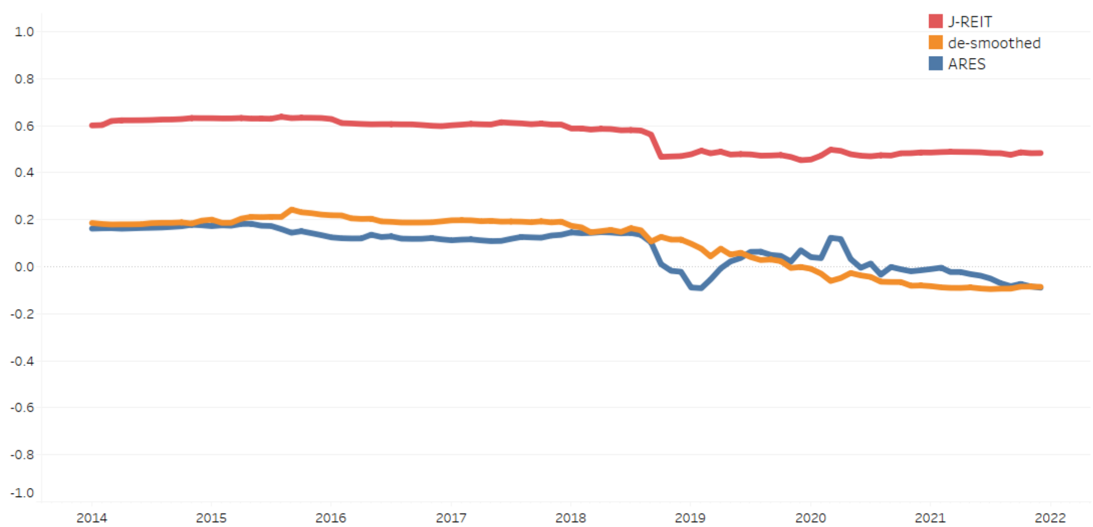
【Figure 3 ：Shier-term volatility (36-month rolling, annualized)】



(Source：Authors' calculations based on FactSet and ARES data)

The long-term correlation coefficients (obtained by rolling basis on the last 10 years of data) are shown in Figure 4. Since the correlation coefficient is the one between the return on real estate and the composite benchmark return, the level varies greatly depending on whether the return on real estate is a market return (J-REIT return) or an appraisal return (ARES return). For "J-REITs," the coefficient is around 0.6 when data in the GFC period is included, and it remains at around 0.5 even when the data in the period is dropped, while for "de-smoothed ARES" and "ARES," it is around 0.2 when data in the GFC period is included, and as the data in the period is dropped out, it drops and remains stable around 0. Therefore, when quantifying the risk of portfolio by its variance, real estate that is traded in private placement will provide greater diversification benefits than J-REIT into a portfolio over the long term.

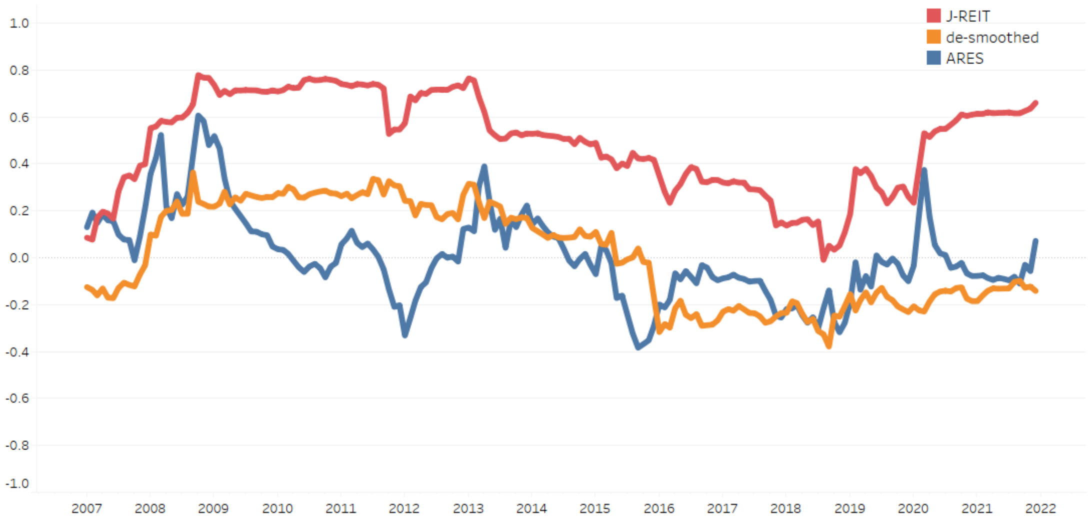
【Figure 4 ：Correlation coefficient with composite BM (120-month rolling)】



(Source：Authors' calculations based on FactSet and ARES data)

The short-term correlation coefficients (obtained on a rolling basis based on the last three years of data) are shown in Figure 5. The short-term correlation coefficients look quite different from the long-term correlation coefficients. Even for the correlation coefficient between the J-REIT return, which is a market return and the composite benchmark return fluctuates widely from 0 to 0.8 in the short term. Therefore, if rebalancing in the short term is assumed, a diversification effect can be expected even for J-REITs depending on timing. It is interesting that while the movement of the correlation coefficient between "de-smoothed ARES" return and the composite benchmark return across 0, which is the level of the long-term correlation coefficient, is similar to that between "J-REIT" return and the composite benchmark return, the movement of the correlation coefficient between "ARES" return and the composite benchmark return is significantly different from that between "J-REIT" return and the composite benchmark return. This is due to the fact that the de-smoothed return is a return obtained by de-smoothing the ARES return and is similar to the J-REIT return in that there is generally no time-series correlation.

【Figure 5: Correlation coefficient with composite BM（36 month rolling)】

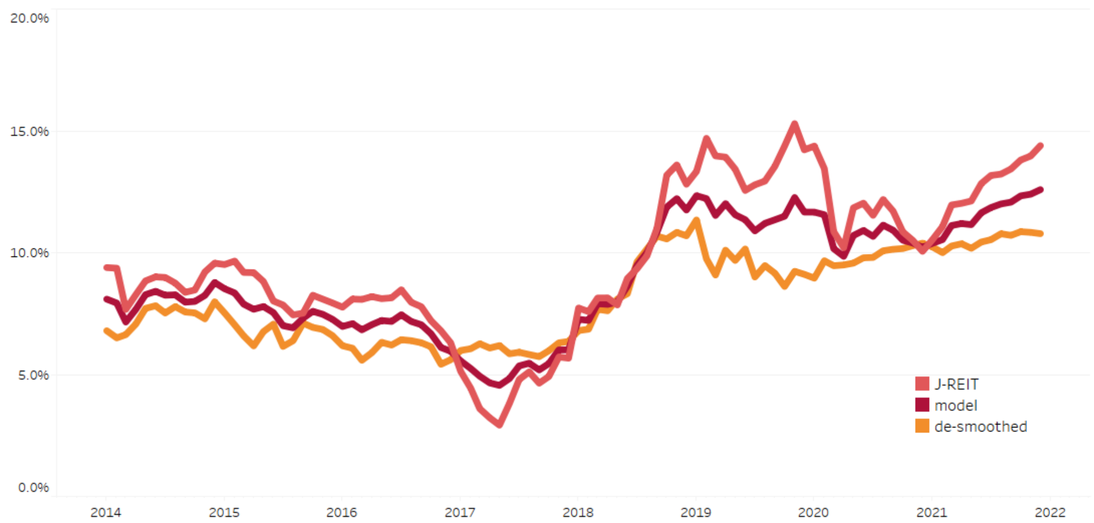


(Source：Authors' calculations based on FactSet and ARES data)

3.3.3 Real estate returns and relative value analysis

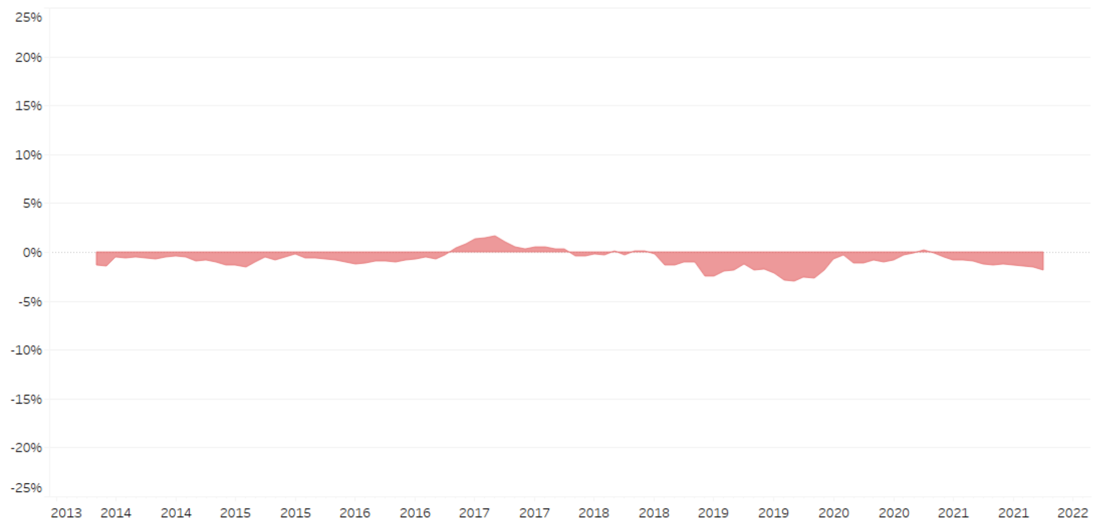
Figure 6 shows rolling averages of annualized monthly returns over the past 10 years for the J-REIT return, the return of the model, and the de-smoothed return. The average returns fluctuate, in order of magnitude, "J-REIT" (3%〜15%), "the model" (5%〜12%), and "de-smoothed ARES" (6%〜11%). Equation (13), which represents the return of the model, is consistent with the results in Figure 6, since it implies the average of the J-REIT return and the de-smoothed return. The rolling 10 years average difference, which is the de-smoothed return minus the return of the model, is shown in Figure 7 as a relative value analysis. The valuation of the model for real estate is in between the valuation in the private placement and the valuation in the capital market and is a fair value for real estate. Positive relative value in Figure 7 means that the de-smoothed return (the return on real estate when traded in private placement) over the past 10 years has outperformed the fair return on real estate and is overvalued within the real estate market. Figure 7 shows that the degree of overvaluation and undervaluation are all generally within about 2% per annum, suggesting that over the long period, the valuation of real estate in private placement and that in the market converge.

【Figure 6: Monthly returns（120-month rolling, annualized)】



(Source：Authors' calculations based on FactSet and ARES data)

【Figure 7: Relative Value Comparison（De-smoothed Returns-Returns for the Model, 120-month rolling)】

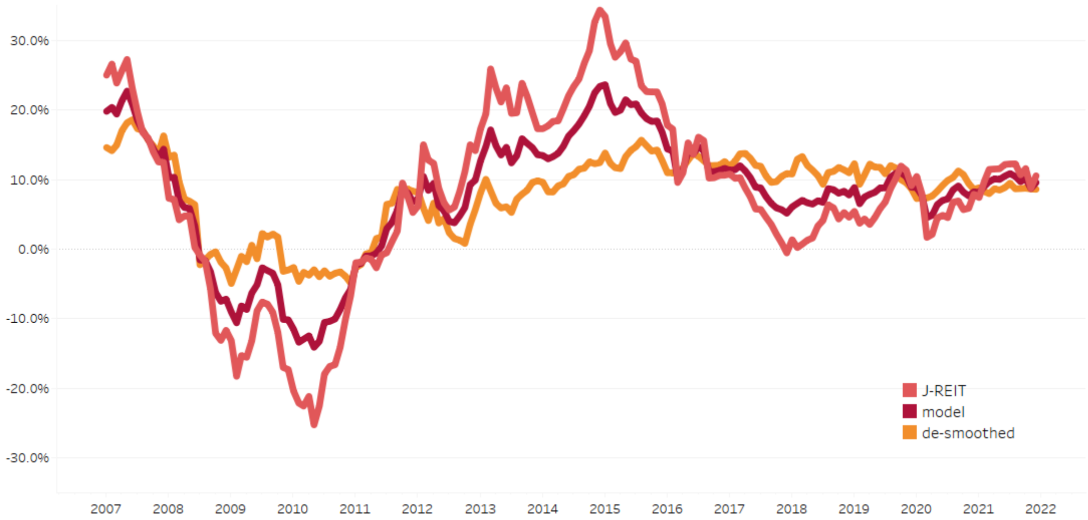


(Source：Authors' calculations based on FactSet and ARES data)

Similarly, for the J-REIT return, the return of the model, and the de-smoothed return, Figure 8 shows the rolling average of the annualized monthly returns over the past three years. The rolling averages fluctuate, in order of magnitude, for the J-REIT（-25%～35%）, the model（-14%～24%）, and the de-smoothed ARES（-5%～15%）. The ranking of the magnitude of fluctuations is the same as that of the past 10 years period and the consistency with equation (13) is also maintained, but the level of volatility is significantly different.

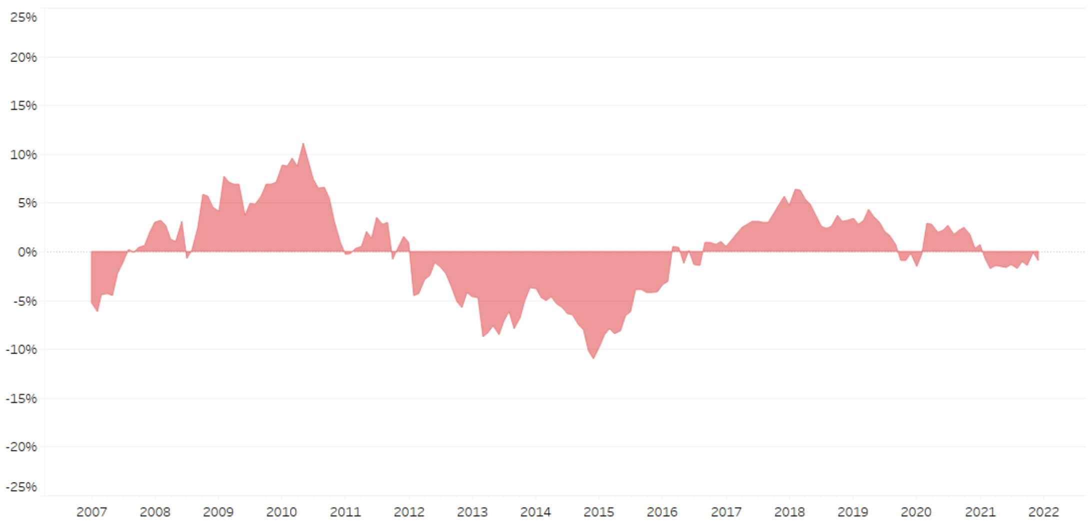
Reflecting this, the relative value analysis shown in Figure 9 shows that the extent of overvaluation or undervaluation ranges from about 11% per year in any case. In other words, it confirms that over a short period of time (the past three years), the valuation of a real estate in the private placement and its valuation in the capital market can diverge significantly.

【Figure 8: Monthly returns（36-month rolling, annualized)】



(Source：Authors' calculations based on FactSet and ARES data)

【Figure 9: Relative Value Comparison（De-smoothed Returns-Returns for the Model, 36-month rolling)】



(Source：Authors' calculations based on FactSet and ARES data)

4. Empirical analysis of portfolio selection

4.1 Data, setup and methods for analysis

The data for real estate used in the empirical analysis of portfolio selection with Sharpe ratio maximization is as described in Section 3.2.1 Data. For the four traditional assets, we use the monthly returns of the following indices: Nomura BPI for domestic bonds, TOPIX for domestic stocks, WGBI for foreign bonds, and ACWI ex Japan for foreign stocks.

The definition of the Sharpe ratio is originally "the value attained by subtracting the risk-free interest rate from the portfolio return and divide it with the standard deviation of the portfolio return." However, since the period under analysis includes long periods of negative short-term interest rates, here the value obtained by assuming that the risk-free interest rate is 0% is called the Sharpe ratio. The portfolio to be analyzed are (1) a portfolio consisting of four traditional assets (domestic bonds, domestic stocks, foreign bonds, and foreign stocks) plus only J-REITs, (2) a portfolio consisting of four traditional assets plus J-REITs and private real estate assuming risk-return characteristics based on ARES returns, and (3) a portfolio consisting of four traditional assets plus J-REITs and private real estate assuming risk-return characteristics based on de-smoothed ARES returns, and (4) a portfolio consisting of four traditional assets plus J-REITs and private real estate assuming risk-return characteristics based on the model.

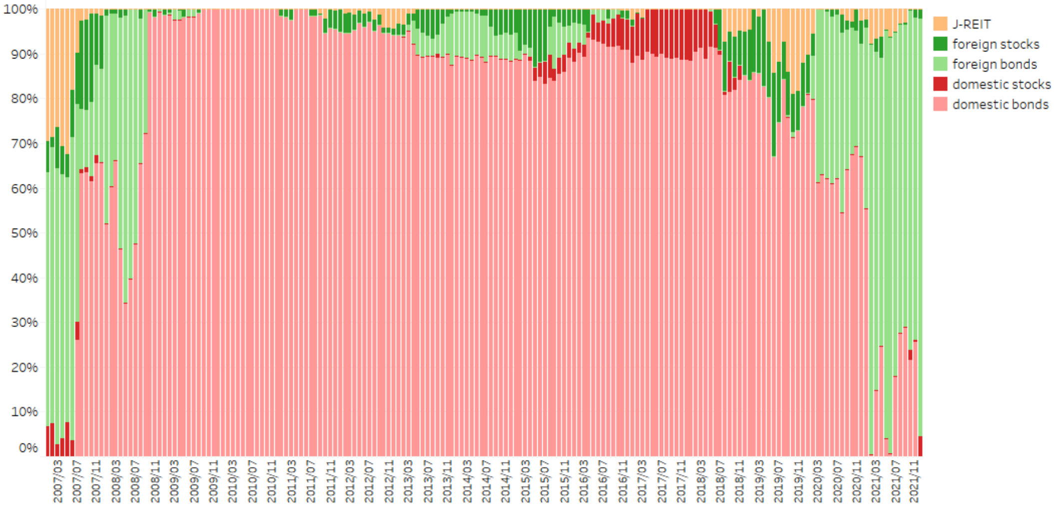
The reason for adopting these four different portfolios in the analysis is that when optimizing a portfolio including real estate in the four traditional assets, if the risk of real estate is simply calculated from ARES returns, the risk is underestimated due to the time series correlation inherent in the ARES returns and real estate is excessively included in the optimal portfolio and it causes practical problem. First, we clarify these problems based on the optimal portfolio in portfolio (2). Next, preceding researches solved the problem to some extent by considering the standard deviation of the de-smoothed real estate return as the real estate risk, and this point is discussed based on the optimal portfolio in portfolio (3).

With respect to real estate traded in private placement, this study proposes a method for deriving the risk of transaction-based return that also takes the J-REIT returns into account in the de-smoothed returns. We discuss the impact of the risk derived from the model on the weight of real estate in the optimal portfolio, based on the optimal portfolio in portfolio (4). Since portfolio (1) is a portfolio that includes only J-REITs in addition to the four traditional assets, by comparing it with portfolio (4), we can grasp the impact of differences in risk characteristics between J-REIT returns and returns of the model, especially differences in correlation coefficients with the four traditional assets, on the weight of real estate in the optimal portfolio.

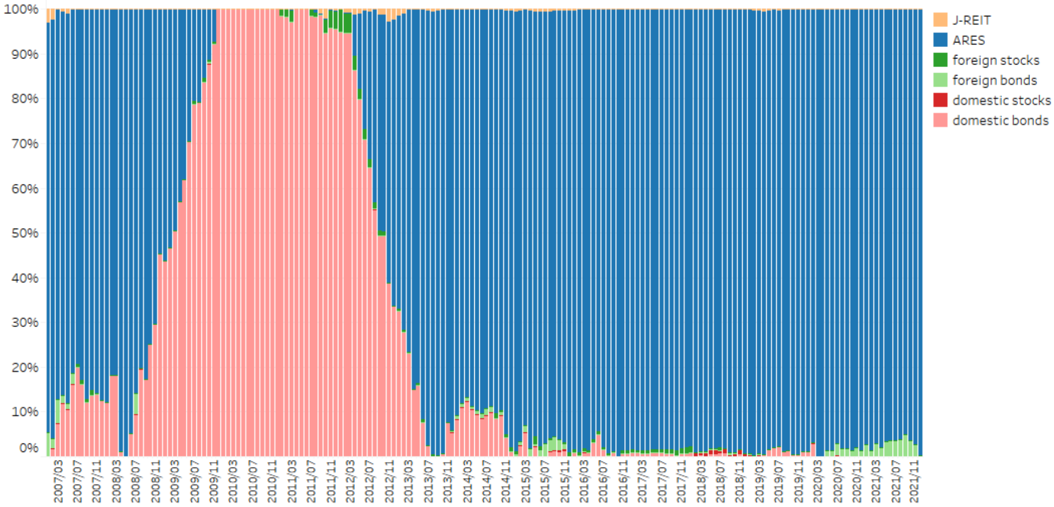
4.2 Empirical Results and Discussion

In portfolio (1) through (4), we attempted to maximize the Sharpe ratio based on risk-return characteristics computed from monthly returns over past three years on a monthly basis. The weight of each asset in the optimal portfolio ("optimal weight") on a monthly basis are shown in Figure 10 through 13 for portfolios (1) through (4), in that order. Figure 11, which shows the optimal weight of portfolio (2), reflects the previously mentioned problem, as it means that almost all weights are allocated to real estate, except for the period from 2009 to 2013.

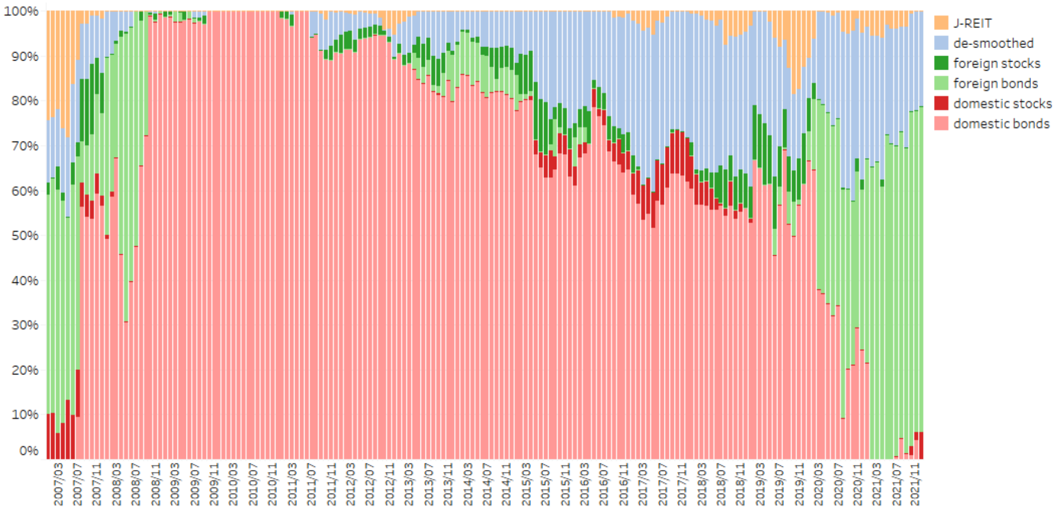
【Figure 10: Optimal weights (traditional assets + J-REITs, 36-month rolling)】



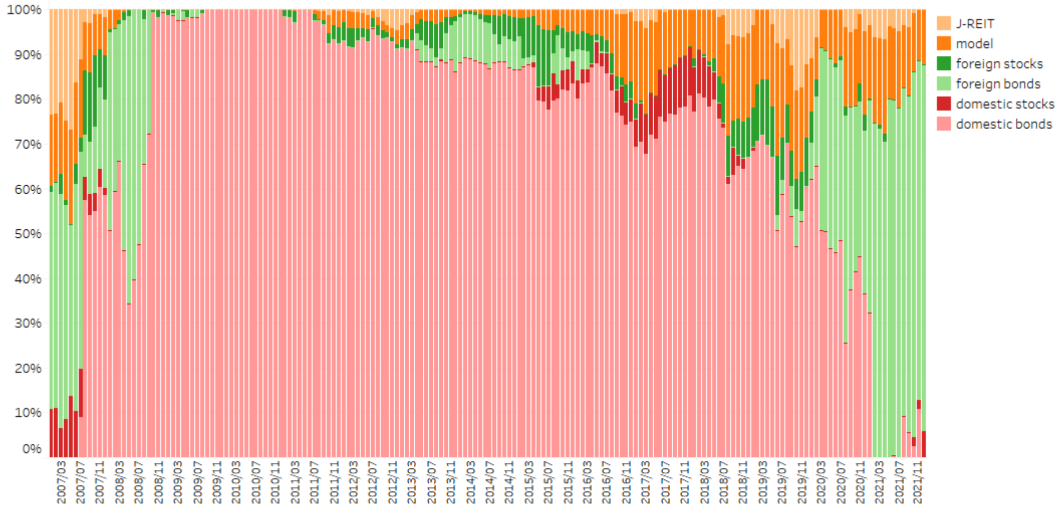
【Figure 11: Optimal weights (traditional assets + J-REITs + ARES, 36-month rolling)】



【Figure 12: Optimal weights (traditional assets+ J-REITs + de-smoothed ARES, 36-month rolling)】



【Figure 13: Optimal weights (traditional assets + J-REITs+ the model, 36-month rolling))】



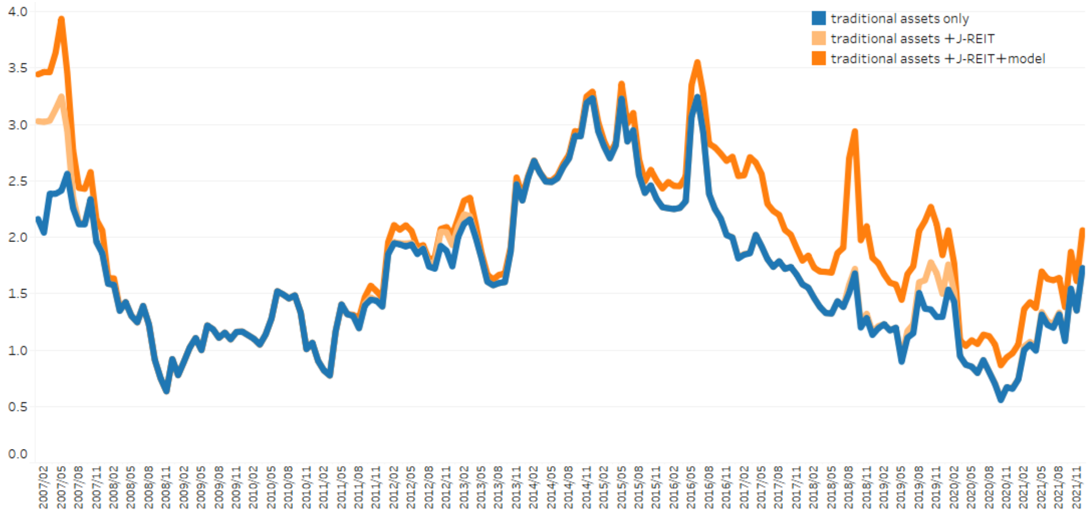
(Source: Figure 10 through 13, all prepared from FactSet and ARES data by authors)

Figure 12, which shows the optimal portfolio in portfolio (3) is closely examined. The optimal portfolio for the period from November 2008 to May 2011, was to hold almost all weights in domestic bonds. Since June 2011, due to the effect of BOJ's zero interest rate policy, the less the domestic bond yield, the less the domestic bond weight in the optimal portfolio and it finally diminished around 0% at December 2020. During the period from June 2011 to December 2019, assets such as domestic stocks, foreign bonds, and foreign stocks were often chosen to replace domestic bonds in the optimal portfolio, but sum of their weights are limited at most to several or around 10 % of total weight. During this period, the consistently increasing weight of private real estate with risk-return characteristic based on de-smoothed ARES returns amounted to about 40% of the optimal portfolio in 2018 when combined with J-REITs, which had a negligible weight, and decreased in weight from 2020. The largest increase in weight since 2020 has been in foreign bonds, which have benefited from the FED's quantitative easing policy. It can also be noticed that the weight of foreign bonds was relatively large in 2007-2008, which corresponds to the period of the BNP Paribas Shock and the GFC. The comparison between Figure 11 and Figure 12 shows that the use of risk-return characteristics based on the de-smoothed ARES return instead of the ARES return is reasonably effective when optimizing a portfolio that includes privately placed real estate.

To examine the impact of risk-return characteristics based on the model on the optimal portfolio, we compare Figure 13, which shows the optimal weights for portfolio (4), with Figure 12. The optimal weights shown in Figure 13 are generally similar to those in Figure 12. A more detailed comparison shows that in Figure 13, private real estate has increased its weight from June 2011 to December 2019, but it is included in the optimal portfolio with 10% to 15% and is slightly less compared to the weight in Figure 12. One possible reason for this is that the risk of privately placed real estate by the model is usually higher than the risk based on de-smoothed ARES return because it reflects the J-REIT returns, which are evaluated by daily market transactions.

We now examine the extent to which adding J-REITs and private real estate to the four traditional assets improves the Sharpe ratio of the optimal portfolio. In Figure 14, we attempt to maximize the Sharpe ratio for three different portfolios, portfolio (1), portfolio (4), and a portfolio consisting of only four traditional assets (5) on a monthly basis, and show the Sharpe ratio of the optimal portfolio in time series. Figure 14 shows that the Sharpe ratio of portfolios (1) and (5) generally overlap, except for some periods in 2007 and 2019. For the period up to July 2015, the Sharpe ratio of portfolio (4) is also generally the same as the other two, with the exception of 2007. However, the Sharpe ratio of portfolio (4) has been higher than the other two since August 2015, with Sharpe ratios ranging from 0.5 to1.5. Thus, adding J-REITs to the four traditional assets would hardly improve the Sharpe ratio, but adding private real estate could improve the Sharpe ratio in some periods.

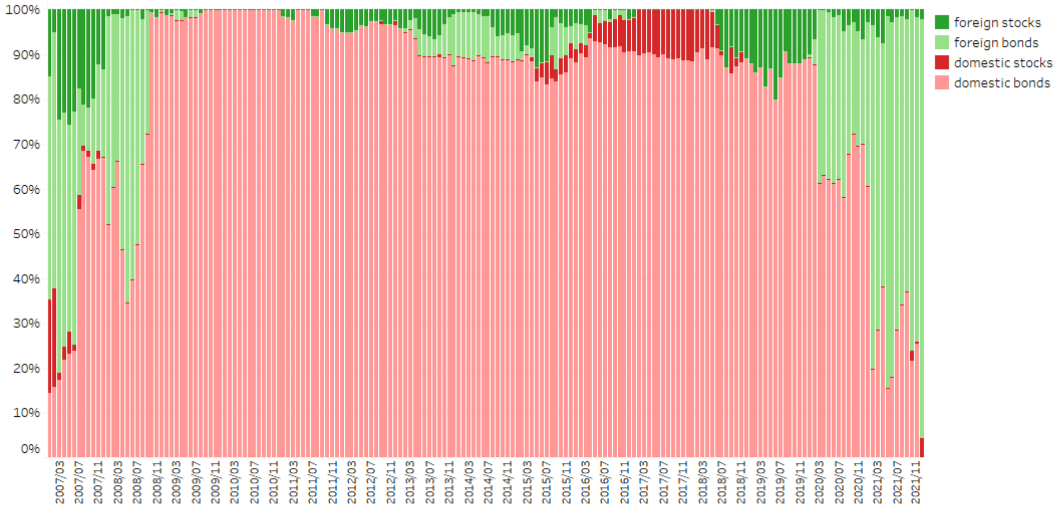
【Figure 14: Sharpe Ratio (36-month rolling)】



(Source: Authors' calculations based on FactSet and ARES data)

This may be due to the low correlation coefficient between the returns of private real estate and the returns of the four traditional assets and the fact that the risk of these assets is usually smaller than that of J-REITs. Here, the optimal portfolio of only the four traditional assets is shown in Figure 15. A closer look at Figure 15 shows that the J-REIT weights in portfolio (1) in Figure 10 are almost replaced by foreign stock for some periods in 2007 and 2019. In other words, in terms of Sharpe ratio maximization, the risk-return characteristics of J-REITs are similar to those of foreign stocks, but they do not exceed the performance of foreign stock for so many periods.

【Figure 15: Optimal weights (traditional assets only, 36-month rolling)】



(Source: Authors' calculations based on FactSet and ARES data)

5. Summary and future researches

In this study, as a research question, we stated, "The prices assumed by real estate appraisers do not necessarily coincide with the prices at which properties are actually traded. Therefore, we cannot regard the de-smoothed ARES return itself as the transaction-based return of privately placed real estate, can we?" To solve this question, we focused on the fact that the J-REIT price is the price at which the real estate is actually traded with the transfer of ownership, and proposed the model to derive mean and standard deviation of transaction-based return by using both the de-smoothed ARES return and the J-REIT return. We also conducted an empirical analysis to understand the characteristics of the model and an empirical analysis of portfolio selection based on Sharpe ratio maximization for a portfolio that includes J-REITs and privately placed real estate in the four traditional assets. Based on the results of the empirical analysis, it was found that the use of the model enables the construction of portfolios that do not excessively include private real estate even if private real estate is included in the four traditional assets, and that in Sharpe ratio maximization, private real estate is included in the optimal portfolio for a larger period than J-REITs, and thus contributes more to building an efficient portfolio.

There are three main issues to be addressed in the future.

The first is to confirm to what extent the model is effective in optimizing the pension fund under the condition of securing a 1.7% real return on pension fund investments (the return on pension fund investments minus the nominal wage growth rate) over the long term with minimal risk, which is a requirement in practice.

The second issue concerns the usage of the model. In the empirical analysis in Section 4, the fair return proposed in the model was used as the return on privately placed real estate in order to eliminate arbitrariness. However, as confirmed in Section 3.3.3, in the short term, the de-smoothed ARES return and the J-REIT return can diverge significantly, and at the same time, both returns can deviate significantly from the fair level. The issue is how to quantify the period until both returns converge to fair levels and the excess return or losses obtained in the process.

Third issue, related to the second issue, is how to connect the model to the Black-Litterman model. If de-smoothed ARES returns are used for privately traded real estate returns as in preceding researches, the Black-Litterman model can be used directly because the short-term optimization is equivalent to the long-term optimization due to the i.i.d. de-smoothed ARES returns. However, when using the model, it will be necessary to specify parameters that represent the investment horizon, and the appropriate setting of such parameters will be a challenge.

Appendix

Derivation of equation (13) through (15)

To simplify the notation, time-series data of the de-smoothed return R_t and the J-REIT return R_t^J are expressed by $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ and $\boldsymbol{y} = (y_1, y_2, \cdots, y_n)$, respectively and define the triad of the parameters as $\boldsymbol{\theta} = (\mu, \sigma^2, \rho)$. The density function of transaction-based return R_t^P for privately traded real estate using the data set (x_i, y_i) is

$$f(x_i, y_i, \boldsymbol{\theta}) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} exp\left\{-\frac{1}{2(1-\rho^2)} Q(x_i, y_i)\right\}, \qquad \text{(A-1-1)}$$

$$Q(x_i, y_i) = \frac{(x_i-\mu)^2}{\sigma^2} - 2\rho\frac{(x_i-\mu)(y_i-\mu)}{\sigma^2} + \frac{(y_i-\mu)^2}{\sigma^2} \qquad \text{(A-1-2)}$$

So, likelihood function $L(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta})$ and the log-likelihood function $l(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta})$ for the time-series data set $\boldsymbol{x}$ and $\boldsymbol{y}$ are

$$L(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta}) = \prod_{i=1}^n f(x_i, y_i, \boldsymbol{\theta}) \qquad \text{(A-3)}$$

$$\begin{aligned} l(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta}) &= logL(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta}) \\ &= -nlog2\pi - nlog(\sigma^2) - \frac{1}{2}nlog(1-\rho^2) - \frac{1}{2(1-\rho^2)\sigma^2}\{\textstyle\sum_{i=1}^n (x_i-\mu)^2 - \\ &\quad 2\rho \textstyle\sum_{i=1}^n (x_i-\mu)(y_i-\mu) + \sum_{i=1}^n (y_i-\mu)^2\} \end{aligned} \qquad \text{(A-4)}$$

Differentiating the log-likelihood function $l(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta})$ with respect to each of the three parameters μ, σ^2, ρ and setting them equal to 0, we obtain three equations to estimate the parameters.

$$\frac{\partial l}{\partial \mu} = -\frac{1}{2(1-\rho^2)\sigma^2}\left\{-\sum_{i=1}^n 2(x_i-\mu) - 2\rho\sum_{i=1}^n \{-(y_i-\mu)-(x_i-\mu)\} - \sum_{i=1}^n 2(y_i-\mu)\right\}$$

$$= \frac{1}{(1-\rho^2)\sigma^2}\{\textstyle\sum_{i=1}^n (x_i-\mu) - \rho\sum_{i=1}^n \{x_i + y_i - 2\mu\} + \sum_{i=1}^n (y_i-\mu)\} = 0 \qquad \text{(A-5)}$$

$$\frac{\partial l}{\partial(\sigma^2)} = -n\frac{1}{(\sigma^2)} + \frac{1}{2(1-\rho^2)(\sigma^2)^2}\left\{\sum_{i=1}^n (x_i-\mu)^2 - 2\rho\sum_{i=1}^n (x_i-\mu)(y_i-\mu) + \sum_{i=1}^n (y_i-\mu)^2\right\} = 0$$

(A-6)

$$\frac{\partial l}{\partial \rho} = -\frac{1}{2}n\frac{-2\rho}{(1-\rho^2)} - \frac{-(-2\rho)}{2\sigma^2(1-\rho^2)^2}\left\{\sum_{i=1}^n (x_i-\mu)^2 - 2\rho\sum_{i=1}^n (x_i-\mu)(y_i-\mu) + \sum_{i=1}^n (y_i-\mu)^2\right\}$$
$$- \frac{1}{2(1-\rho^2)\sigma^2}\left\{-2\rho\sum_{i=1}^n (x_i-\mu)(y_i-\mu)\right\}$$

$$= \frac{n\rho}{(1-\rho^2)} - \frac{\rho}{\sigma^2(1-\rho^2)^2}\{\textstyle\sum_{i=1}^n (x_i-\mu)^2 - 2\rho\sum_{i=1}^n (x_i-\mu)(y_i-\mu) + \sum_{i=1}^n (y_i-\mu)^2\} -$$

$$\frac{1}{2(1-\rho^2)\sigma^2}\{-2\sum_{i=1}^{n}(x_i-\mu)(y_i-\mu)\}=0 \qquad \text{(A-7)}$$

Multiplying equation (A-5) by $(1-\rho^2)\sigma^2$ and dividing n, we obtain

$$\frac{1}{n}\{\sum_{i=1}^{n}(x_i-\mu)-\rho\sum_{i=1}^{n}\{x_i+y_i-2\mu\}+\sum_{i=1}^{n}(y_i-\mu)\}=0$$

Putting $\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$ and $\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i$, we are able to transform above equation to

$$\bar{x}-\mu-\rho(\bar{x}+\bar{y}-2\mu)+\bar{y}-\mu=0$$

and we finally obtain

$$\mu=\frac{1}{2}(\bar{x}+\bar{y}) \qquad \text{(A-8)}$$

Equation (A-8) is the same as equation (13).

Next, we arrange $\{\cdot\}$ of equation (A-6).

$$\begin{aligned}
&\sum_{i=1}^{n}(x_i-\mu)^2-2\rho\sum_{i=1}^{n}(x_i-\mu)(y_i-\mu)+\sum_{i=1}^{n}(y_i-\mu)^2\\
&=\sum_{i=1}^{n}\{(x_i-\bar{x})+(\bar{x}-\mu)\}^2-2\rho\sum_{i=1}^{n}\{(x_i-\bar{x})+(\bar{x}-\mu)\}\{(y_i-\bar{y})+(\bar{y}-\mu)\}+\sum_{i=1}^{n}\{(y_i-\bar{y})+(\bar{y}-\mu)\}^2\\
&=\sum_{i=1}^{n}(x_i-\bar{x})^2+2(\bar{x}-\mu)\sum_{i=1}^{n}(x_i-\bar{x})+n(\bar{x}-\mu)^2-2\rho\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})-2\rho\sum_{i=1}^{n}(x_i-\bar{x})(\bar{y}-\mu)-2\rho\sum_{i=1}^{n}(\bar{x}-\mu)(y_i-\bar{y})-2\rho n(\bar{x}-\mu)(\bar{y}-\mu)+\sum_{i=1}^{n}(y_i-\bar{y})^2+2(\bar{y}-\mu)\sum_{i=1}^{n}(y_i-\bar{y})+n(\bar{y}-\mu)^2
\end{aligned}$$

Dividing $\{\cdot\}$ in equation (A-6) by n, we obtain $\frac{1}{n}\{\cdot\}$ and arrange it using equation (A-8),

$$\begin{aligned}
&\frac{1}{n}\{\sum_{i=1}^{n}(x_i-\mu)^2-2\rho\sum_{i=1}^{n}(x_i-\mu)(y_i-\mu)+\sum_{i=1}^{n}(y_i-\mu)^2\}\\
&=\sigma_x^2+(\bar{x}-\mu)^2-2\rho\rho_{x,y}\sigma_x\sigma_y-2\rho(\bar{x}-\mu)(\bar{y}-\mu)+\sigma_y^2+(\bar{y}-\mu)^2\\
&=\sigma_x^2+\left(\frac{\bar{x}-\bar{y}}{2}\right)^2-2\rho\rho_{x,y}\sigma_x\sigma_y-2\rho\left(\frac{\bar{x}-\bar{y}}{2}\right)\left(\frac{\bar{y}-\bar{x}}{2}\right)+\sigma_y^2+\left(\frac{\bar{y}-\bar{x}}{2}\right)^2\\
&=\sigma_x^2-2\rho\rho_{x,y}\sigma_x\sigma_y+\sigma_y^2+\frac{1}{2}(1+\rho)(\bar{x}-\bar{y})^2 \qquad \text{(A-9)}
\end{aligned}$$

In the process of deriving equation (A-9) above,

$$\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)(y_i-\mu)=\rho_{x,y}\sigma_x\sigma_y-\frac{1}{4}(\bar{x}-\bar{y})^2 \qquad \text{(A-10)}$$

can also be obtained.

Multiplying both sides of equation (A-6) by $\frac{\sigma^2}{n}$, and substituting equation (A-9), we obtain

$$-1+\frac{1}{2(1-\rho^2)\sigma^2}\left\{\sigma_x^2-2\rho\rho_{x,y}\sigma_x\sigma_y+\sigma_y^2+\frac{1}{2}(1+\rho)(\bar{x}-\bar{y})^2\right\}=0$$

Then arranging this, we obtain

$$\sigma^2=\frac{1}{2(1-\rho^2)}\left\{\sigma_x^2-2\rho\rho_{x,y}\sigma_x\sigma_y+\sigma_y^2+\frac{1}{2}(1+\rho)(\bar{x}-\bar{y})^2\right\} \qquad \text{(A-11)}$$

Also, by multiplying both sides of equation (A-7) by $\frac{(1-\rho^2)}{n\rho}$ and substituting equation (A-9) and equation (A-10), we obtain

$$1-\frac{1}{(1-\rho^2)\sigma^2}\left\{\sigma_x^2-2\rho\rho_{x,y}\sigma_x\sigma_y+\sigma_y^2+\frac{1}{2}(1+\rho)(\bar{x}-\bar{y})^2\right\}+\frac{1}{\rho\sigma^2}\left\{\rho_{x,y}\sigma_x\sigma_y-\frac{1}{4}(\bar{x}-\bar{y})^2\right\}=0$$

Arranging above equation with the fact that the second term on the left-hand side is equal to 2 using equation (A-11), we obtain

$$\rho\sigma^2 = \rho_{x,y}\sigma_x\sigma_y - \frac{1}{4}(\bar{x}-\bar{y})^2 \qquad \text{(A-12)}$$

Now, multiplying both sides of equation (A-11) by $2(1-\rho^2)$ and substituting equation (A-12) after arranging the right-hand side, we obtain

$$2(1-\rho^2)\sigma^2 = \sigma_x^2 - 2\rho\rho_{x,y}\sigma_x\sigma_y + \sigma_y^2 + \frac{1}{2}(1+\rho)(\bar{x}-\bar{y})^2 = \sigma_x^2 + \sigma_y^2 + \frac{1}{2}(\bar{x}-\bar{y})^2 -$$

$$2\rho\left(\rho_{x,y}\sigma_x\sigma_y - \frac{1}{4}(\bar{x}-\bar{y})^2\right) = \sigma_x^2 + \sigma_y^2 + \frac{1}{2}(\bar{x}-\bar{y})^2 - 2\rho^2\sigma^2$$

and this is simplified to

$$\sigma^2 = \frac{1}{2}\left(\sigma_x^2 + \sigma_y^2\right) + \frac{1}{4}(\bar{x}-\bar{y})^2 \qquad \text{(A-13)}$$

Equation (A-13) is the same as equation (14). Substituting this into equation (A-12) and rearranging it, we obtain

$$\rho = \frac{\rho_{x,y}\sigma_x\sigma_y - \frac{1}{4}(\bar{x}-\bar{y})^2}{\frac{1}{2}(\sigma_x^2+\sigma_y^2) + \frac{1}{4}(\bar{x}-\bar{y})^2} \qquad \text{(A-14)}$$

Equation (A-14) is the same as equation (15).

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